CHAPTER 2

Basic Mathematics used in Physics

Quadratic Equation

Roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $x_1 + x_2 = -\frac{b}{a}$;

Product of roots $x_1 x_2 = \frac{c}{a}$

For real roots, $b^2 - 4ac \geq 0$

For imaginary roots, $b^2 - 4ac < 0$

Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots$$

$$(1-x)^n = 1 - nx + \frac{n(n-1)}{2}x^2 - \frac{n(n-1)(n-2)}{6}x^3 + \ldots$$

If $x << 1$ then $(1+x)^n \approx 1 + nx$ & $(1-x)^n \approx 1 - nx$

Logarithm

$\log_{10} N = x \Rightarrow 10^x = N$

$\log_b N = \log_b a * \log_a N$

$\log_b 1 = 0$, $\log_a a = 1$

$\log mn = \log m + \log n$ $\quad$ $\log \frac{m}{n} = \log m - \log n$

$\log m^n = n \log m$ $\quad$ $\log_e m = 2.303 \log_{10} m$

$\log 2 = 0.3010$ $\quad$ $\log 3 = 0.4771$

Componendo and dividendo theorem

If $\frac{p}{q} = \frac{a}{b}$ then $\frac{p+q}{p-q} = \frac{a+b}{a-b}$

Arithmetic progression-AP

$a, a+d, a+2d, a+3d, \ldots a+(n-1)d$

here d = common difference

Sum of n terms $S_n = \frac{n}{2}[2a+(n-1)d]$

n^{th} term, $a_n = a + (n-1)d$

Note: (i) $1+2+3+4+5 \ldots + n = \frac{n(n+1)}{2}$

(ii) $1^2+2^2+3^2+\ldots+ n^2 = \frac{n(n+1)(2n+1)}{6}$

(iii) $1^3+2^2+3^2+\ldots+ n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Geometrical progression-GP

$a, ar, ar^2, ar^3, \ldots$ here, r = common ratio

n^{th} term, $a_n = a.r^{n-1}$

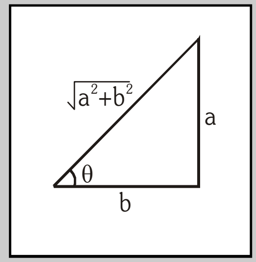
Sum of n terms $S_n = \frac{a(1-r^n)}{1-r}$

Sum of ∞ terms $S_\infty = \frac{a}{1-r}$ [where $|r| < 1$]

KUSAL ACADEMY

TRIGONOMETRY

2π radian = $360^\circ \Rightarrow 1$ rad = 57.3°



$\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$ | $\cos\theta = \dfrac{\text{base}}{\text{hypotenuse}}$ | $\tan\theta = \dfrac{\text{perpendicular}}{\text{base}}$

$\cot\theta = \dfrac{\text{base}}{\text{perpendicular}}$ | $\sec\theta = \dfrac{\text{hypotenuse}}{\text{base}}$ | $\text{cosec}\,\theta = \dfrac{\text{hypotenuse}}{\text{perpendicular}}$

$\sin\theta = \dfrac{a}{\sqrt{a^2+b^2}}$ | $\cos\theta = \dfrac{b}{\sqrt{a^2+b^2}}$ | $\tan\theta = \dfrac{a}{b}$

$\text{cosec}\theta = \dfrac{1}{\sin\theta}$ | $\sec\theta = \dfrac{1}{\cos\theta}$ | $\cot\theta = \dfrac{1}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$ | $1 + \tan^2\theta = \sec^2\theta$ | $1 + \cot^2\theta = \text{cosec}^2\theta$

$\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$ $\qquad$ $\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$

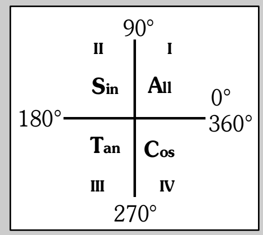
$\tan(A\pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$ $\qquad$ $\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ $\qquad$ $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$ $\qquad$ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

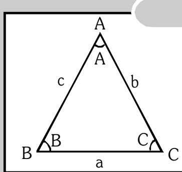
$2\cos A\cos B = \cos(A-B) + \cos(A+B)$ $\qquad$ $2\sin A\cos B = \sin(A+B) + \sin(A-B)$



θ	0° (0)	30° (π/6)	45° (π/4)	60° (π/3)	90° (π/2)	120° (2π/3)	135° (3π/4)	150° (5π/6)	180° (π)	270° (3π/2)	360° (2π)
sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	**0**
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	0	**1**
tanθ	**0**	$\frac{1}{\sqrt{3}}$	**1**	$\sqrt{3}$	∞	$-\sqrt{3}$	**−1**	$-\frac{1}{\sqrt{3}}$	**0**	∞	**0**

$\sin(90^\circ+\theta) = \cos\theta$	$\sin(180^\circ-\theta) = \sin\theta$	$\sin(-\theta) = -\sin\theta$	$\sin(90^\circ-\theta) = \cos\theta$
$\cos(90^\circ+\theta) = -\sin\theta$	$\cos(180^\circ-\theta) = -\cos\theta$	$\cos(-\theta) = \cos\theta$	$\cos(90^\circ-\theta) = \sin\theta$
$\tan(90^\circ+\theta) = -\cot\theta$	$\tan(180^\circ-\theta) = -\tan\theta$	$\tan(-\theta) = -\tan\theta$	$\tan(90^\circ-\theta) = \cot\theta$
$\sin(180^\circ+\theta) = -\sin\theta$	$\sin(270^\circ-\theta) = -\cos\theta$	$\sin(270^\circ+\theta) = -\cos\theta$	$\sin(360^\circ-\theta) = -\sin\theta$
$\cos(180^\circ+\theta) = -\cos\theta$	$\cos(270^\circ-\theta) = -\sin\theta$	$\cos(270^\circ+\theta) = \sin\theta$	$\cos(360^\circ-\theta) = \cos\theta$
$\tan(180^\circ+\theta) = \tan\theta$	$\tan(270^\circ-\theta) = \cot\theta$	$\tan(270^\circ+\theta) = -\cot\theta$	$\tan(360^\circ-\theta) = -\tan\theta$

sine law



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

For small θ

$\sin\theta \approx \theta$ $\qquad$ $\cos\theta \approx 1$ $\qquad$ $\tan\theta \approx \theta$ $\qquad$ $\sin\theta \approx \tan\theta$

cosine law

$$\cos A = \frac{b^2+c^2-a^2}{2bc},\ \cos B = \frac{c^2+a^2-b^2}{2ca},\ \cos C = \frac{a^2+b^2-c^2}{2ab}$$

Differentiation

- $y = x^n \to \frac{dy}{dx} = nx^{n-1}$ • $y = \ell nx \to \frac{dy}{dx} = \frac{1}{x}$
- $y = \sin x \to \frac{dy}{dx} = \cos x$ • $y = \cos x \to \frac{dy}{dx} = -\sin x$
- $y = e^{\alpha x+\beta} \to \frac{dy}{dx} = \alpha e^{\alpha x+\beta}$ • $y = uv \to \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
- $y = f(g(x)) \Rightarrow \frac{dy}{dx} = \frac{df(g(x))}{dg(x)} \times \frac{d(g(x))}{dx}$
- y=k(constant) $\Rightarrow \frac{dy}{dx} = 0$
- $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Integration

C = Arbitrary constant, k = constant

- $\int f(x)dx = g(x) + C$
- $\frac{d}{dx}(g(x)) = f(x)$
- $\int kf(x)dx = k\int f(x)dx$
- $\int (u+v+w)dx = \int udx + \int vdx + \int wdx$
- $\int e^x dx = e^x + C$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
- $\int \frac{1}{x}dx = \ell nx + C$
- $\int \sin xdx = -\cos x + C$
- $\int \cos xdx = \sin x + C$
- $\int e^{\alpha x+\beta}dx = \frac{1}{\alpha}e^{\alpha x+\beta} + C$
- $\int (\alpha x+\beta)^n dx = \frac{(\alpha x+\beta)^{n+1}}{\alpha(n+1)} + C$

Definite integration

$$\int_a^b f(x)dx = |g(x)|_a^b = g(b) - g(a)$$

Area under the curve y = f(x) from x =a to a = b is

$$A = \int_a^b f(x)dx$$

Maxima & Minima of a function y=f(x)

- For maximum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = -ve$
- For minimum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = +ve$

Average of a varying quantity

If y = f(x) then $\langle y \rangle = \bar{y} = \frac{\int_{x_1}^{x_2} ydx}{\int_{x_1}^{x_2} dx} = \frac{\int_{x_1}^{x_2} ydx}{x_2 - x_1}$

FORMULAE FOR DETERMINATION OF AREA

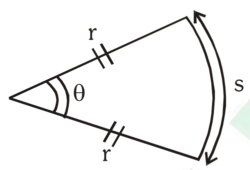
- Area of a square = $(side)^2$
- Area of rectangle = length $\times$ breadth
- Area of a triangle = $\frac{1}{2}\times$base$\times$height
- Area of a trapezoid

  $= \frac{1}{2}\times$ (distance between parallel sides) $\times$ (sum of parallel sides)
- Area enclosed by a circle = πr^2 (r = radius)
- Surface area of a sphere = $4\pi r^2$ (r = radius)
- Area of a parallelogram = base $\times$ height
- Area of curved surface of cylinder = $2\pi r\ell$

  where r = radius and ℓ = length
- Area of whole surface of cylinder = $2\pi r(r+\ell)$ where ℓ = length
- Area of ellipse = πab

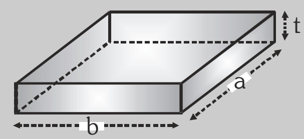
  (a & b are semi major and semi minor axis respectively)
- Surface area of a cube = $6(side)^2$
- Total surface area of a cone = $\pi r^2 + \pi r\ell$

  where $\pi r\ell = \pi r\sqrt{r^2+h^2}$ = lateral area
- Arc length $s = r.\theta$
- Area of sector = $\frac{r^2\theta}{2}$



- Plane angle, $\theta = \frac{s}{r}$ radian
- Solid angle, $\Omega = \frac{A}{r^2}$ steradian

FORMULAE FOR DETERMINATION OF VOLUME



- Volume of a rectangular slab

  = length $\times$ breadth $\times$ height

  = abt
- Volume of a cube = $(side)^3$
- Volume of a sphere = $\frac{4}{3}\pi r^3$

  (r = radius)
- Volume of a cylinder = $\pi r^2\ell$

  (r = radius and ℓ = length)
- Volume of a cone = $\frac{1}{3}\pi r^2 h$

  (r = radius and h = height)

KEY POINTS

- To convert an angle from degree to radian, we have to multiply it by $\frac{\pi}{180°}$ and to convert an angle from radian to degree, we have to multiply it by $\frac{180°}{\pi}$.
- By help of differentiation, if y is given, we can find $\frac{dy}{dx}$ and by help of integration, if $\frac{dy}{dx}$ is given, we can find y.
- The maximum and minimum values of function

  $A\cos\theta + B\sin\theta$ are $\sqrt{A^2+B^2}$ and $-\sqrt{A^2+B^2}$ respectively.
- $(a+b)^2 = a^2 + b^2 + 2ab$ $(a-b)^2 = a^2 + b^2 - 2ab$

  $(a+b)(a-b) = a^2 - b^2$ $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

  $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

VECTORS

Vector Quantities

A physical quantity which requires magnitude and a particular direction, when it is expressed.
Parallel vectors – two vectors having same direction.

antiparallel vectors – vectors in opposite direction.

Equal vectors – Vectors which have equal magnitude and same direction
Negative or opposite vectors – Vectors having equal magnitude but opposite direction.

Null vector or Zero vector

A vector having zero magnitude. The direction of a zero vector is indeterminate.

$$\vec{A} + (-\vec{A}) = \vec{0}$$

Unit vector

A vector having unit magnitude. It is used to speciafy direction.

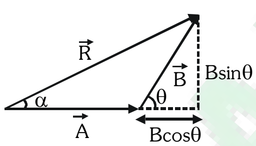
Unit vector in direction of $\vec{A}$, $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

Triangle law of Vector addition

$\boxed{\vec{R} = \vec{A} + \vec{B}}$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$

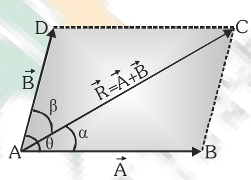


If A = B then $R = 2A\cos\dfrac{\theta}{2}$ & $\alpha = \dfrac{\theta}{2}$

$R_{max} = A + B$ for $\theta = 0°$; $R_{min} = A \sim B$ for $\theta = 180°$

Parallelogram Law of Addition of Two Vectors

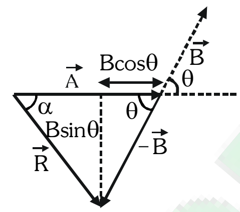
If two vectors are represented by two adjacent sides of a parallelogram which are directed away from their common point then their sum (i.e. resultant vector) is given by the diagonal of the parallelogram passing away through that common point.



$$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC} = \vec{R} \text{ or } \vec{A} + \vec{B} = \vec{R} \Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} \text{ and } \tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

Vector subtraction



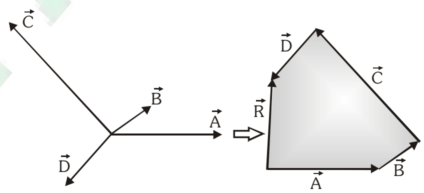
$$\boxed{\vec{R} = \vec{A} - \vec{B}} \Rightarrow \vec{R} = \vec{A} + (-\vec{B})$$

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}, \quad \tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

If A = B then $R = 2A\sin\dfrac{\theta}{2}$

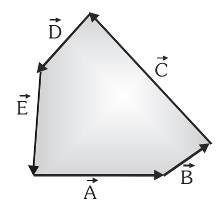
Addition of More than Two Vectors (Law of Polygon)

If some vectors are represented by sides of a polygon in same order, then their resultant vector is represented by the closing side of polygon in the opposite order.



$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

In a polygon if head of the last vector coincide with the tail of the first vectors, in other words vectors are forming closed polygon, then their resultant is null vector.



$$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} = 0$$

Rectangular component of a 3–D vector

- $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

  Angle made with x-axis

  $$\cos\alpha = \frac{A_x}{A} = \frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = \ell$$

  Angle made with y-axis

  $$\cos\beta = \frac{A_y}{A} = \frac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = m$$

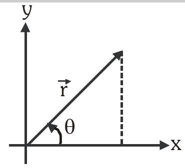
Angle made with z-axis

$$\cos\gamma = \frac{A_z}{A} = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = n$$

- ℓ, m, n are called direction cosines
  $\ell^2+m^2+n^2=\cos^2\alpha+\cos^2\beta+\cos^2\gamma$

$$= \frac{A_x^2 + A_y^2 + A_z^2}{\left(\sqrt{A_x^2 + A_y^2 + A_z^2}\right)^2} = 1 \text{ or } \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

General Vector in x-y plane



$$\vec{r} = x\hat{i} + y\hat{j} = r(\cos\theta\hat{i} + \sin\theta\hat{j})$$

EXAMPLES :

1. Construct a vector of magnitude 6 units making an angle of 60° with x-axis.

Sol. $\vec{r} = r(\cos 60\hat{i} + \sin 60\hat{j}) = 6\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) = 3\hat{i} + 3\sqrt{3}\hat{j}$

2. Construct an unit vector making an angle of 135° with x axis.

Sol. $\hat{r} = 1(\cos 135°\hat{i} + \sin 135°\hat{j}) = \frac{1}{\sqrt{2}}(-\hat{i} + \hat{j})$

Multiplication of a vector by a number

If $\vec{b} = k_x\vec{a}$ then magnitude of $\vec{b}$ is k times $|\vec{a}|$, and direction of $\vec{b}$ is same as $\vec{a}$

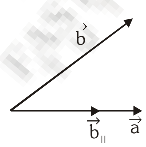
Scalar product (Dot Product)

- $\vec{A}.\vec{B} = AB\cos\theta \Rightarrow$ Angle between two vectors $\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$

- If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ & $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ then
  $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$ and angle between $\vec{A}$ & $\vec{B}$ is given by

$$\cos\theta = \frac{\vec{A}.\vec{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

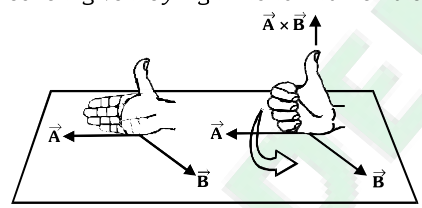
- $\hat{i}.\hat{i} = 1$, $\hat{j}.\hat{j} = 1$, $\hat{k}.\hat{k} = 1$, $\hat{i}.\hat{j} = 0$, $\hat{i}.\hat{k} = 0$, $\hat{j}.\hat{k} = 0$
- Component of vector $\vec{b}$ along vector $\vec{a}$,
  $\vec{b}_{\parallel} = (\vec{b}.\hat{a})\hat{a}$



- Component of $\vec{b}$ perpendicular to $\vec{a}$,
  $\vec{b}_{\perp} = \vec{b} - \vec{b}_{\parallel} = \vec{b} - (\vec{b}\cdot\hat{a})\hat{a}$

Cross Product (Vector product)

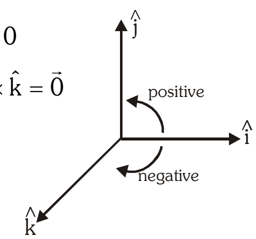
- $\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$ where $\hat{n}$ is a vector perpendicular to $\vec{A}$ & $\vec{B}$ or their plane and its direction given by right hand thumb rule.



- $$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - B_xA_z) + \hat{k}(A_xB_y - B_xA_y)$$

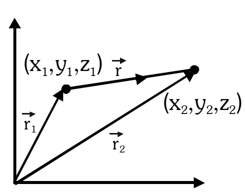
- $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
- $(\vec{A} \times \vec{B}).\vec{A} = (\vec{A} \times \vec{B}).\vec{B} = 0$
- $\hat{i} \times \hat{i} = \vec{0}$, $\hat{j} \times \hat{j} = \vec{0}$, $\hat{k} \times \hat{k} = \vec{0}$
- $\hat{i} \times \hat{j} = \hat{k}$; $\hat{j} \times \hat{k} = \hat{i}$,
  $\hat{k} \times \hat{i} = \hat{j}$; $\hat{j} \times \hat{i} = -\hat{k}$
  $\hat{k} \times \hat{j} = -\hat{i}$, $\hat{i} \times \hat{k} = -\hat{j}$



Differentiation

- $$\frac{d}{dt}(\vec{A}.\vec{B}) = \frac{d\vec{A}}{dt}.\vec{B} + \vec{A}.\frac{d\vec{B}}{dt}$$
- $$\frac{d}{dt}(\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$$

When a particle moved from (x_1, y_1, z_1) to (x_2, y_2, z_2) then its displacement vector

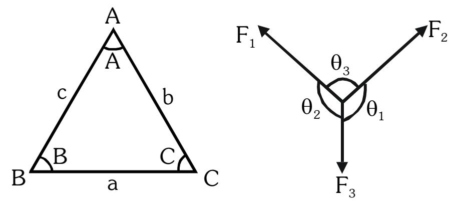


$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Magnitude: $r = |\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

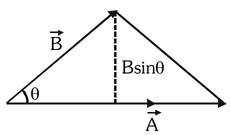
Lami's theorem



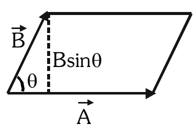
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

Area of triangle

$$\text{Area} = \frac{|\vec{A} \times \vec{B}|}{2} = \frac{1}{2} AB\sin\theta$$



Area of parallelogram



$$\text{Area} = |\vec{A} \times \vec{B}| = AB\sin\theta$$

For parallel vectors

$$\vec{A} \times \vec{B} = \vec{0}$$

For perpendicular vectors

$$\vec{A}.\vec{B} = 0$$

For coplanar vectors

$$\vec{A}.(\vec{B} \times \vec{C}) = 0$$

If A,B,C points are collinear

$$\overrightarrow{AB} = \lambda\overrightarrow{BC}$$

Examples of dot products

- Work, $W = \vec{F}.\vec{d} = Fd\cos\theta$ where $F \to$ force, $d \to$ displacement
- Power, $P = \vec{F}.\vec{v} = Fv\cos\theta$ where $F \to$ force, $v \to$ velocity
- Electric flux, $\phi_E = \vec{E}.\vec{A} = EA\cos\theta$ where $E \to$ electric field, $A \to$ Area
- Magnetic flux, $\phi_B = \vec{B}.\vec{A} = BA\cos\theta$ where $B \to$ magnetic field, $A \to$ Area
- Potential energy of dipole in uniform field, $U = -\vec{p}.\vec{E}$ where $p \to$ dipole moment, where $E \to$ Electric field

Examples of cross products

- Torque $\vec{\tau} = \vec{r} \times \vec{F}$ where $r \to$ position vector, $F \to$ force
- Angular momentum $\vec{J} = \vec{r} \times \vec{p}$ where $r \to$ position vector, $p \to$ linear momentum
- Linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$ where $r \to$ position vector, $\omega \to$ angular velocity
- Torque on dipole placed in electric field $\vec{\tau} = \vec{p} \times \vec{E}$

  where $p \to$ dipole moment, $E \to$ electric field

KEY POINTS

- Tensor : A quantity that has different values in different directions is called tensor.

  Example : Moment of Inertia

  In fact tensors are merely a generalisation of scalars and vectors; a scalar is a zero rank tensor, and a vector is a first rank tensor.
- Electric current is not a vector as it does not obey the law of vector addition.
- A unit vector has no unit.
- To a vector only a vector of same type can be added and the resultant is a vector of the same type.
- A scalar or a vector can never be divided by a vector.