LAWS OF MOTION & FRICTION

FORCE

A push or pull that one object exerts on another.

Forces in nature

There are four fundamental forces in nature :
- Gravitational force
- Electromagnetic force
- Strong nuclear force
- Weak force

Types of forces on macroscopic objects

(a) Field Forces or Range Forces :

These are the forces in which contact between two objects is not necessary.

Ex. (i) Gravitational force between two bodies.
(ii) Electrostatic force between two charges.

(b) Contact Forces :

Contact forces exist only as long as the objects are touching each other.

Ex. (i) Normal force. (ii) Frictional force

(c) Attachment to Another Body :

Tension (T) in a string and spring force (F = kx) comes in this group.

NEWTON'S FIRST LAW OF MOTION (or Galileo's law of Inertia)

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external unbalanced force to change that state.

Inertia : Inertia is the property of the body due to which body opposes the change of it's state. Inertia of a body is measured by mass of the body.

$$\boxed{\text{inertia} \propto \text{mass}}$$

Newton's second law

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

(Linear momentum $\vec{p} = m\vec{v}$)

For constant mass system $\vec{F} = m\vec{a}$

Momentum

It is the product of the mass and velocity of a body

i.e. momentum $\vec{p} = m\vec{v}$

- **SI Unit** : kg m s^{-1}
- **Dimensions** : $[M L T^{-1}]$

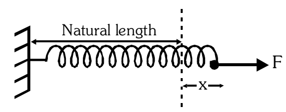
Newton's third law of motion :

Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.

i.e. $\vec{F}_{A/B} = -\vec{F}_{B/A}$

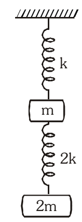
Spring Force (According to Hooke's law)

In equilibrium F=kx (k is spring constant)



Note : Spring force is non impulsive in nature.

Ex. If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring.

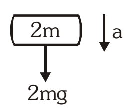


Sol. Initial stretches $x_{upper} = \frac{3mg}{k}$

and $x_{lower} = \frac{mg}{k}$

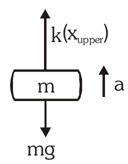
On cutting the lower spring, by virtue of non–impulsive nature of spring the stretch in upper spring remains same immediately after cutting the spring. Thus,

Lower block :



$2mg = 2ma \Rightarrow a = g$

Upper block :



$k\left(\frac{3mg}{k}\right) - mg = ma \Rightarrow a = 2g$

Motion of bodies in contact

When two bodies of masses m_1 and m_2 are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called force of contact. These two bodies will move with same acceleration a

(i) When the force F acts on the body with mass m_1 as shown in figure (i) : $F = (m_1 + m_2)a$

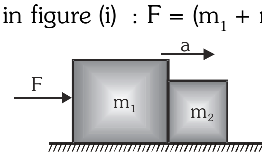


Fig.(1) : When the force F acts on mass m_1

If the force exerted by m_2 on m_1 is f_1 (force of contact) then for body m_1: $(F - f_1) = m_1a$

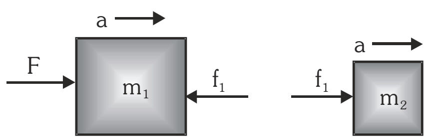


Fig. 1(a) : F.B.D. representation of action and reaction forces.

For body m_2 **:**

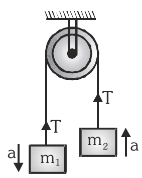
$f_1 = m_2a \Rightarrow$ action of m_1 on m_2: $f_1 = \dfrac{m_2F}{m_1 + m_2}$

Pulley system

A single fixed pulley changes the direction of force only and in general, assumed to be massless and frictionless.

SOME CASES OF PULLEY

Case – I :



Let $m_1 > m_2$ now for mass m_1, $m_1 g - T = m_1a$

for mass m_2, $T - m_2 g = m_2 a$

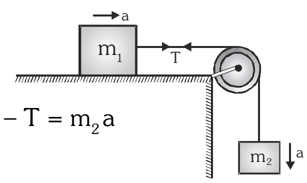
Acceleration $= a = \dfrac{(m_1 - m_2)}{(m_1 + m_2)} g = \dfrac{\text{net pulling force}}{\text{total mass to be pulled}}$

Tension $= T = \dfrac{2m_1m_2}{(m_1 + m_2)} g = \dfrac{2 \times \text{Product of masses}}{\text{Sum of two masses}} g$

Reaction at the suspension of pulley :

$$R = 2T = \frac{4m_1m_2 g}{(m_1 + m_2)}$$

Case – II



For mass m_1 :

$T = m_1 a$

For mass m_2 : $m_2g - T = m_2a$

Acceleration:

$a = \dfrac{m_2g}{(m_1 + m_2)}$ and $T = \dfrac{m_1m_2}{(m_1 + m_2)} g$

FRAME OF REFERENCE

- **Inertial frames of reference :** A reference frame which is either at rest or in uniform motion along the straight line. A non–accelerating frame of reference is called an inertial frame of reference.

  All the fundamental laws of physics have been formulated in respect of inertial frame of reference.
- **Non–inertial frame of reference :** An accelerating frame of reference is called a non–inertial frame of reference. Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force.

Pseudo force:

The force on a body due to acceleration of non–inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$, where $\vec{a}_0$ is acceleration of non–inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pseudo force must be opposite to the direction of acceleration of the non–inertial frame.

When we draw the free body diagram of a mass, with respect to an **inertial frame of reference** we apply only the real forces (forces which are actually acting on the mass). But when the free body diagram is drawn from a non–inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

$\sum \vec{F}_{real} + \vec{F}_{pseudo} = m\vec{a}$ (where $\vec{a}$ is acceleration of object in non inertial reference frame) & $\vec{F}_{pseudo} = -m\vec{a}_a$

(where $\vec{a}_0$ is acceleration of non inertial reference frame).

Man in a Lift

(a) If the lift moving with constant velocity v upwards or downwards. In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift.

So apparent weight $W' = Mg$ = Actual weight.

(b) If the lift is accelerated upward with constant acceleration a. Then forces acting on the man w.r.t. observed inside the lift are

(i) Weight $W = Mg$ downward

(ii) Fictitious force $F_0 = Ma$ downward.

So apparent weight $W' = W + F_0 = Mg + Ma = M(g+a)$

(c) If the lift is accelerated downward with acceleration $a < g$.

Then w.r.t. observer inside the lift fictitious force $F_0 = Ma$ acts upward while weight of man $W = Mg$ always acts downward.

So apparent weight

$W' = W - F_0 = Mg - Ma = M(g-a)$

Special Case :

If $a = g$ then $W' = 0$ (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

(d) If lift accelerates downward with acceleration $a > g$. Then as in Case (c).

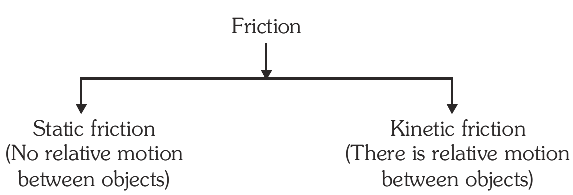
Apparent weight $W' = M(g-a)$ is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

FRICTION

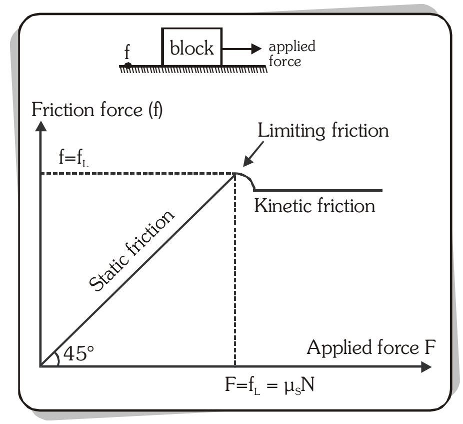
Friction is the force of two surfaces in contact, or the force of a medium acting on a moving object. (i.e. air on aircraft.)

Frictional forces arise due to molecular interactions. In some cases friction acts as a supporting force and in some cases it acts as opposing force.

- **Cause of Friction:** Friction arises on account of strong atomic or molecular forces of attraction between the two surfaces at the point of actual contact.
- **Types of friction**



Graph between applied force and force of friction



- **Static friction coefficient**

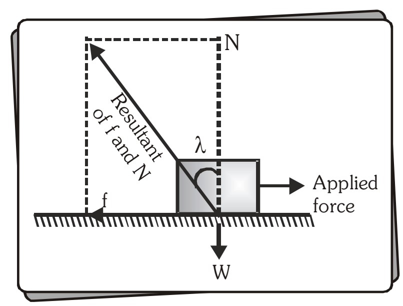
$$\mu_s = \frac{(f_s)_{max}}{N}, \quad 0 \le f_s \le \mu_s N, \quad \vec{f}_s = -\vec{F}_{applied}$$

$$(f_s)_{max} = \mu_s N = \text{limiting friction}$$

- **Kinetic friction coefficient**

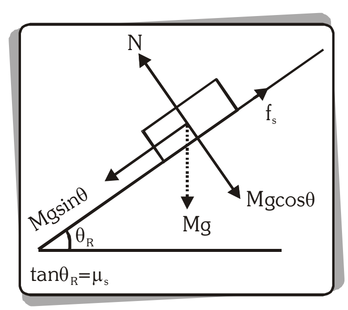
$$\mu_k = \frac{f_k}{N}, \vec{f}_k = -(\mu_k N)\hat{v}_{rel}$$

- **Angle of Friction (λ)**



$$\tan\lambda = \frac{f_S}{N} = \frac{\mu_s N}{N} = \mu_S$$

- **Angle of repose** : The maximum angle of an inclined plane for which a block remains stationary on the plane.



Dependent Motion of Connected Bodies

Method I : Method of constraint equations

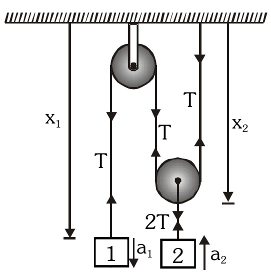
$$\sum x_i = \text{constant} \Rightarrow \sum \frac{dx_i}{dt} = 0 \Rightarrow \sum \frac{d^2x_i}{dt^2} = 0$$

- For n moving bodies we have $x_1, x_2, \ldots x_n$
- No. of constraint equations = no. of strings

Method II : Method of virtual work :

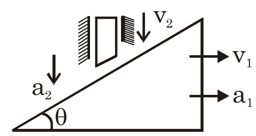
The sum of scalar products of tension forces applied by connecting links of constant length and displacement of corresponding contact points equal to zero.

$$\sum \vec{T} \cdot \vec{x} = 0 \Rightarrow \sum \vec{T} \cdot \vec{v} = 0 \Rightarrow \sum \vec{T} \cdot \vec{a} = 0$$



Here $2a_2 = a_1$

Normal constraint : displacements, velocities & accelerations of both objects should be same along C.N.



e.g. $a_2 = a_1 \tan\theta$ & $v_2 = v_1 \tan\theta$

KEY POINTS

- Aeroplanes always fly at low altitudes because according to Newton's III law of motion as aeroplane displaces air & at low altitude density of air is high.
- Rockets move by pushing the exhaust gases out so they can fly at low & high altitude.
- Pulling (figure I) is easier than pushing (figure II) on a rough horizontal surface because normal reaction is less in pulling than in pushing.

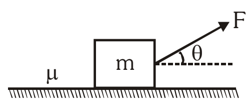


Fig. I

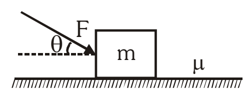


Fig. II

- While walking on ice, one should take small steps to avoid slipping. This is because smaller step increases the normal reaction and that ensure smaller friction.