CIRCULAR MOTION

Definition of Circular Motion

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point. That fixed point is called centre and the distance is called radius of circular path.

Radius Vector :

The vector joining the centre of the circle and the center of the particle performing circular motion is called radius vector. It has constant magnitude and variable direction. It is directed outwards.

$\vec{k} \cdot \vec{v} = 0$ & $\vec{r}$ & $\vec{v}$ always in same plane.

Frequency (n) :

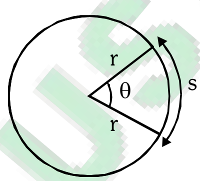
No. of revolutions described by particle per sec. is its frequency. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.)

Time Period (T) :

It is time taken by particle to complete one revolution.

$$T = \frac{1}{n}$$

- Angle $\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$



- Average angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$ (a scalar quantity)
- Instantaneous angular velocity

$$\omega = \frac{d\theta}{dt} \text{ (a vector quantity)}$$

- For uniform angular velocity $\omega = \frac{2\pi}{T} = 2\pi f$ or $2\pi n$
- Angular displacement $\theta = \omega t$

  $\omega \rightarrow$ Angular frequency n or f = frequency
- Relation between ω and v $\omega = \frac{v}{r}$

In vector form velocity $\vec{v} = \vec{\omega} \times \vec{r}$

- Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$

  $= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} = \vec{a}_t + \vec{a}_C$

- Tangential acceleration: $a_t = \frac{dv}{dt} = \alpha r$

$$\left[\vec{a}_t = \text{component of } \vec{a} \text{ along } \vec{v} = (\vec{a} \cdot \hat{v})\hat{v} = \left(\frac{dv}{dt}\right)\hat{v}\right]$$

- Centripetal acceleration :

$$a_C = \omega v = \frac{v^2}{r} = \omega^2 r \text{ or } \vec{a}_C = \omega^2 r(-\hat{r})$$

$$\vec{a}_C \cdot \vec{v} = 0$$

- Magnitude of net acceleration :

$$a = \sqrt{a_C^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

For uniform circular motion $\frac{d|\vec{v}|}{dt} = 0 = a_t$

- If a is contant, then following equations hold

(i) $\Delta\theta = \theta_f - \theta_i$

(ii) $\omega_f = \omega_i + \alpha t$

(iii) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

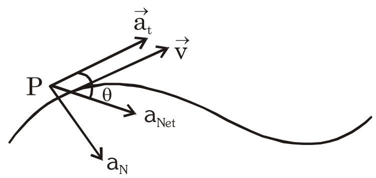
(iv) $\omega_f^2 = \omega_i^2 + 2\alpha\theta$

(v) $\theta = \omega_f t - \frac{1}{2}\alpha t^2$

(vi) $\theta = \frac{(\omega_i + \omega_f)t}{2}$

(vii) $\alpha = \left(\frac{\omega_f - \omega_i}{t}\right)$

Curvilinear Motion :



$\vec{a}_{Net} = \frac{d\vec{v}}{dt}$;

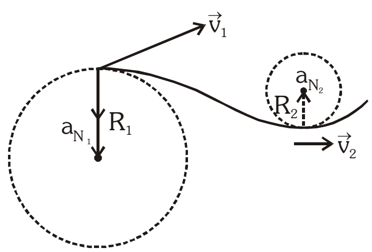
components of $\vec{a}_{Net}$ along $\vec{v} = \vec{a}_t$

components of $\vec{a}_{Net}$ perpendicular to $\vec{v} = \vec{a}_N$

$\vec{a}_T = \frac{d|\vec{v}|}{dt}$; a_N is responsible for change of direction

Radius of Curvature :

$a_N = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_N}$



$R_1 > R_2$; Radius of curvature doesn't remains constant
R is a property of curves, not of the particle
(If a bee follows this path instead of the particle then its radius of curvature will be the same)

Maximum speed of in circular motion :

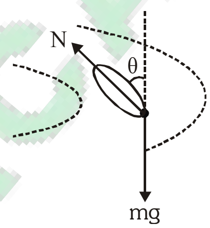
- On unbanked road : $v_{max} = \sqrt{\mu_s Rg}$
- On banked road :

$$v_{max} = \sqrt{\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)Rg} = \sqrt{\tan(\theta + \phi)Rg}$$

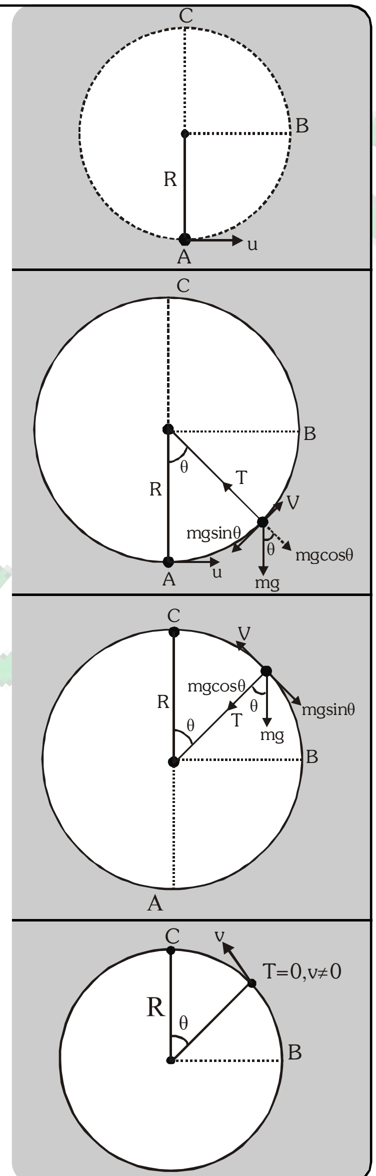
$$v_{min} = \sqrt{Rg\tan(\theta - \phi)}\ ;\ v_{min} \le v_{car} \le v_{max}$$

where ϕ = angle of friction = $\tan^{-1}\mu_s$; θ = angle of banking

- Bending of cyclist : $\tan\theta = \frac{v^2}{rg}$



Circular motion in vertical plane



A. Condition to complete vertical circle $u \geq \sqrt{5gR}$

If $u = \sqrt{5gR}$ then Tension at C is equal to 0 and tension at A is equal to 6mg

Velocity at B: $v_B = \sqrt{3gR}$

Velocity at C: $v_C = \sqrt{gR}$

From A to B : $T = mg\cos\theta + \frac{mv^2}{R}$

From B to C : $T = \frac{mv^2}{R} - mg\cos\theta$

B. Condition for pendulum motion (oscillating condition)

$u \leq \sqrt{2gR}$ (in between A to B)

Velocity can be zero but T never be zero between A & B.

Because T is given by $T = mg\cos\theta + \frac{mv^2}{R}$

C. Condition for leaving path : $\sqrt{2gR} < u < \sqrt{5gR}$

Particle crosses the point B but not complete the vertical circle.

Tension will be zero in between B to C & the angle where T = 0

$\cos\theta = \frac{u^2 - 2gR}{3gR}$; θ is from vertical line

Note : After leaving the circle, the particle will follow a parabolic path.

* T is maximum at the bottom & minimum at the top.

KEY POINTS

- Average angular velocity is a scalar physical quanity whereas instantaneous angular velocity is a vector physical quanity.
- Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity.

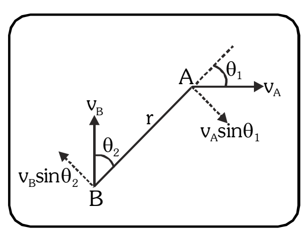
$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1$

But $\vec{\theta}_1 + \vec{\theta}_2 \neq \vec{\theta}_2 + \vec{\theta}_1$

- **Relative Angular Velocity**

Relative angular velocity of a particle 'A' w.r.t. other moving particle B is the angular velocity of the position vector of A w.r.t. B.

That means it is the rate at which position vector of 'A' w.r.t. B rotates at that instant



$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{seperation between A and B}}$$

here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2$ $\therefore \omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$