KUSAL ACADEMY

WORK, ENERGY & POWER

WORK DONE

$$W = \int dW = \int \vec{F}.d\vec{r} = \int F dr \cos\theta$$

[where θ is the angle between $\vec{F}$ & $d\vec{r}$]

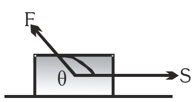
- For constant force $W = \vec{F}.\vec{d} = Fd\cos\theta$
- For Unidirectional force

$W = \int dW = \int F dx =$ Area between F–x curve and x-axis.

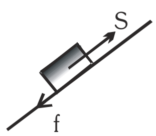
NATURE OF WORK DONE

Although work done is a scalar quantity, yet its value may be positive, negative or even zero

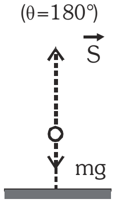
Negative work



($\theta > 90°$)

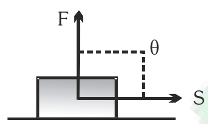


Work done by friction force ($\theta = 180°$)

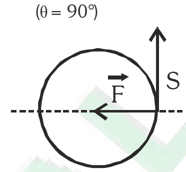


Work done by gravity ($\theta = 180°$)

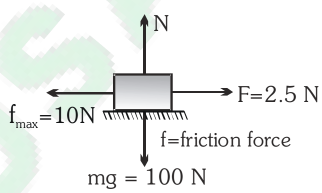
Zero work



($\theta = 90°$)

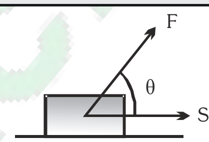


Motion of particle on circular path (uniform) ($\theta = 90°$)

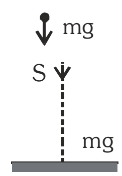


As f = F, hence S = 0

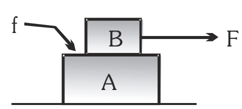
Positive work



($\theta < 90°$)



Motion under gravity ($\theta = 0°$)



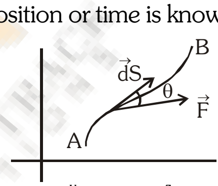
Work done by friction force on block A ($\theta = 0°$)

WORK DONE BY VARIABLE FORCE

A force varying with position or time is known as the variable force

$\vec{F} = F x\hat{i} + F y\hat{j} + F z\hat{k}$

$d\vec{S} = dx\hat{i} + dy\hat{j} + dz\hat{k}$



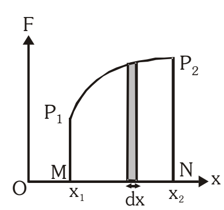
$$W_{AB} = \int_A^B \vec{F}\cdot d\vec{S} = \int_{x_A}^{x_B} F x dx + \int_{y_A}^{y_B} F y dy + \int_{z_A}^{z_B} F z dz$$

Calculation of work done from force–displacement graph :

Total work done,

$$W = \sum_{x_1}^{x_2} F dx$$

$= $ Area of P_1P_2NM



Kinetic energy

- The energy possessed by a body by virtue of its motion is called kinetic energy.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(\vec{v}.\vec{v})$$

- Kinetic energy is a frame dependent quantity.

Work energy theorem $(W = \Delta KE)$

Change in kinetic energy = work done by all force

For conservative force $F(x) = -\frac{dU}{dx}$

Change in potential energy $\Delta U = -\int F(x)dx$

Conservative Forces

- Work done does not depend upon path.
- Work done in a round trip is zero.
- Central forces, spring forces etc. are conservative forces
- When only a conservative force acts within a system, the kinetic energy and potential energy can change into each other. However, their sum, the mechanical energy of the system, doesn't change.
- Work done is completely recoverable.
- If $\vec{F}$ is a conservative force then $\vec{\nabla} \times \vec{F} = \vec{0}$ (i.e. curl of $\vec{F}$ is zero)

Non-conservative Forces

- Work done depends upon path.
- Work done in a round trip is not zero.
- Forces are velocity-dependent & retarding in nature e.g. friction, viscous force etc.
- Work done against a non-conservative force may be dissipated as heat energy.
- Work done is not recoverable.

POTENTIAL ENERGY

- The energy possessed by a body by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Potential energy of a body at any position in a conservative force field is defined as the external work done against the action of conservative force in order to shift it from a certain reference point (PE = 0) to the present position.
- Potential energy of a body in a conservative force field is equal to the work done by the conservative force in moving the body from its present position to reference position.
- At a certain reference position, the potential energy of the body is assumed to be zero or the body is assumed to have lost the capacity of doing work.
- Relationship between conservative force field and potential energy :

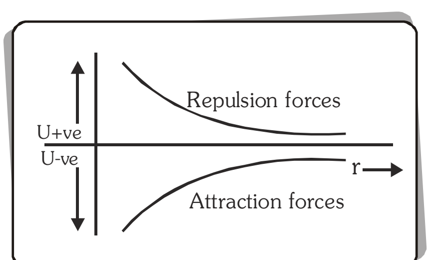
$$\vec{F} = -\nabla U = -\text{grad}(U) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

- If force varies with only one dimension (say along x-axis) then

$$F = -\frac{dU}{dx} \Rightarrow dU = -Fdx \Rightarrow \int_{U_1}^{U_2} dU = -\int_{x_1}^{x_2} Fdx$$

$$\Rightarrow \boxed{\Delta U = -W_C}$$

- Potential energy may be positive or negative or even zero



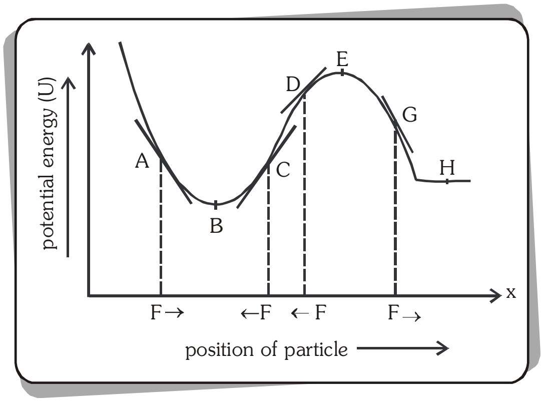
i) Potential energy is positive, if force field is repulsive in nature

ii) Potential energy is negative, if force field is attractive in nature

- If r ↑ (separation between body and force centre), U ↑, force field is attractive or vice-versa.
- If r ↑, U ↓, force field is repulsive in nature.

Potential energy curve and equilibrium

It is a curve which shows the change in potential energy with the position of a particle.



Stable Equilibrium :

After a particle is slightly displaced from its equilibrium position if it tends to come back towards equilibrium then it is said to be in stable equilibrium.

At point **A :** slope $\frac{dU}{dx}$ is negative so F is positive

At point **C :** slope $\frac{dU}{dx}$ is positive. so F is negative

$$\text{At equilibrium} \quad F = -\frac{dU}{dx} = 0$$

At point **B :** it is the point of stable equilibrium.

At point **B :** $U = U_{min}$, $\frac{dU}{dx} = 0$ & $\frac{d^2U}{dx^2}$ = positive

Unstable equilibrium :

After a particle is slightly displaced from its equilibrium position, if it tends to move away from equilibrium position then it is said to be in unstable equilibrium.

At point **D** : slope $\frac{dU}{dx}$ is positive so F is negative ; At point **G** : slope $\frac{dU}{dx}$ is negative so F is positive

At point **E** : it is the point of unstable equilibrium;

At point **E** $U=U_{max}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ = negative

Neutral equilibrium

After a particle is slightly displaced from its equilibrium position if no force acts on it then the equilibrium is said to be neutral equilibrium.

Point **H** corresponds to neutral equilibrium $\Rightarrow U = \text{constant}$; $\frac{dU}{dx}=0, \frac{d^2U}{dx^2}=0.$

Law of conservation of Mechanical energy

Total mechanical (kinetic + potential) energy of a system remains constant if only conservative forces are acting on the system of particles or the work done by all other forces is zero. From work energy theorem $W = \Delta KE$

Proof : For internal conservative forces $W_{int} = -\Delta U$

So $W=W_{ext} + W_{int} = 0 + W_{int} = -\Delta U \Rightarrow -\Delta U = \Delta KE$
$\Rightarrow \Delta(KE+U) = 0 \Rightarrow KE+U=\text{constant}$

- Spring force $F=-kx$, Elastic potential energy stored in spring $U(x) = \frac{1}{2}kx^2$
- Mass and energy are equivalent and are related by $E = mc^2$

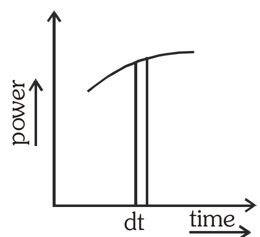
Power

- Power is a scalar quantity with dimension $M^1L^2T^{-3}$
- SI unit of power is J/s or watt
- 1 horsepower = 746 watt = 550 ft–lb/sec.

Average power : $P_{av} = W/t$

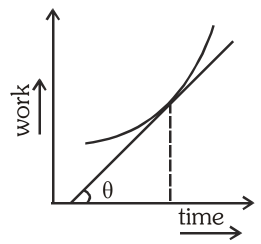
Instantaneous power : $P = \frac{dW}{dt} = \frac{\vec{F}.d\vec{r}}{dt} = \vec{F}.\vec{v}$

fig.(a)



$W = \int P dt$

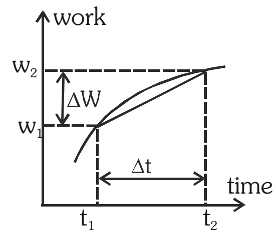
fig.(b)



instantaneous power

$P = \frac{dW}{dt} = \tan\theta$

fig.(c)



average power

$\bar{P} = P_{av} = \frac{W_2 - W_1}{t_2 - t_1} = \frac{\Delta W}{\Delta t}$

- For a system of varying mass $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$
- If $\vec{v}$ = constant then $\vec{F} = \vec{v}\frac{dm}{dt}$ then $P = \vec{F}.\vec{v} = v^2\frac{dm}{dt}$
- In rotatory motion : $P = \tau\frac{d\theta}{dt} = \tau\omega$
- Efficiency $\eta = \frac{\text{Output Energy}}{\text{Input Energy}}$

KEY POINTS

- A body may gain kinetic energy and potential energy simultaneously because principle of conservation of mechanical energy may not be valid every time.
- Comets move around the sun in elliptical orbits. The gravitational force on the comet due to sun is not normal to the comet's velocity but the work done by the gravitational force is zero in complete round trip because gravitational force is a conservative force.
- Work done by static friction may be positive because static friction may acts along the direction of motion of an object.