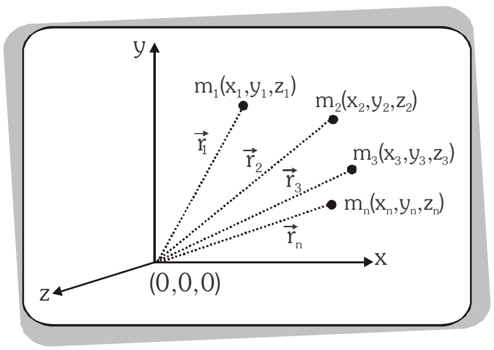
Centre of mass :

Centre of mass of system is the point associated with the system which have same acceleration as the acceleration of point mass (of same mass as that of system) would have under the application of same external force.

Centre of mass of system of discrete particles



Total mass of the body :

$M = m_1 + m_2 + ... + m_n$ then

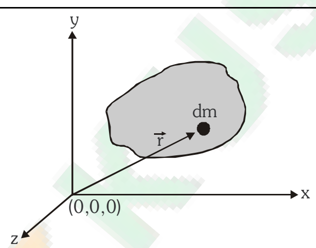
$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{1}{M}\Sigma m_i\vec{r}_i$$

co-ordinates of centre of mass :

$$x_{cm} = \frac{1}{M}\Sigma m_i x_i\,,\; y_{cm} = \frac{1}{M}\Sigma m_i y_i \;\&\; z_{cm} = \frac{1}{M}\Sigma m_i z_i$$

Centre of mass of continuous distribution of particles

$$\vec{R}_{CM} = \frac{1}{M}\int \vec{r}dm$$



$$x_{cm} = \frac{1}{M}\int x\,dm\,,\; y_{cm} = \frac{1}{M}\int y\,dm \text{ and } z_{cm} = \frac{1}{M}\int z\,dm$$

x, y, z are the co-ordinate of the COM of the dm mass.

The centre of mass after removal of a part of a body

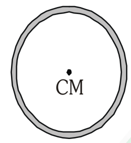
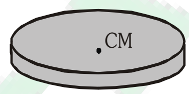
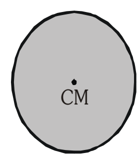
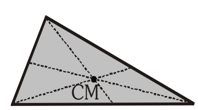
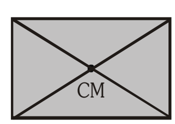
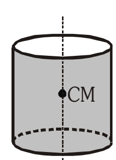
Original mass (M) – mass of the removed part (m)

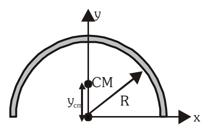
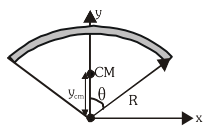
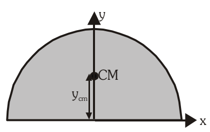
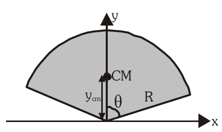
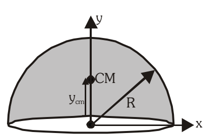
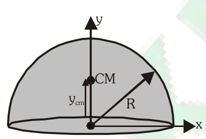
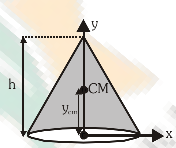
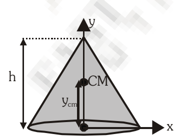
= {original mass (M)}+{–mass of the removed part (m)}

The formula changes to:

$$x_{CM} = \frac{Mx - mx'}{M - m}\,;\; y_{CM} = \frac{My - my'}{M - m}\,;\; y_{CM} = \frac{Mz - mz'}{M - m}$$

CENTRE OF MASS OF SOME COMMON OBJECTS

Body	Shape of body	Position of centre of mass
Uniform Ring		Centre of ring
Uniform Disc		Centre of disc
Uniform Rod		Centre of rod
Solid sphere/ hollow sphere		Centre of sphere
Triangular plane lamina		Point of intersection of the medians of the triangle i.e. centroid
Plane lamina in the form of a square or rectangle or parallelogram		Point of intersection of diagonals
Hollow/solid cylinder		Middle point of the axis of cylinder

Body	Shape of body	Position of centre of mass
Half ring		$y_{cm} = \frac{2R}{\pi}$
Segement of a ring		$y_{cm} = \frac{R\sin\theta}{\theta}$
Half disc (plate)		$y_{cm} = \frac{4R}{3\pi}$
Sector of a disc (plate)		$y_{cm} = \frac{2R\sin\theta}{3\theta}$
Hollow hemisphere		$y_{cm} = \frac{R}{2}$
Solid hemisphere		$y_{cm} = \frac{3R}{8}$
Hollow cone		$y_{cm} = \frac{h}{3}$
Solid cone		$y_{cm} = \frac{h}{4}$

MOTION OF CENTRE OF MASS

For a system of particles,
velocity of centre of mass

$$\vec{v}_{CM} = \frac{d\vec{R}_{CM}}{dt} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots}{m_1 + m_2 + \ldots.}$$

Similarly acceleration

$$\vec{a}_{CM} = \frac{d}{dt}\left(\vec{v}_{CM}\right) = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots}{m_1 + m_2 + \ldots.}$$

Law of conservation of linear momentum

Linear momentum of a system of particles is equal to the product of mass of the system with velocity of its centre of mass.

From Newton's second law $\vec{F}_{ext.} = \frac{d\left(M\vec{v}_{CM}\right)}{dt}$

If $\vec{F}_{ext.} = \vec{0}$ then $M\vec{v}_{CM}$ = constant

If no external force acts on a system the velocity of its centre of mass remains constant, i.e., velocity of centre of mass is unaffected by internal forces.

Impulse – Momentum theorem

Impulse of a force is equal to the change of momentum

$$\int_{t_1}^{t_2} \vec{F}dt = \Delta\vec{p}$$

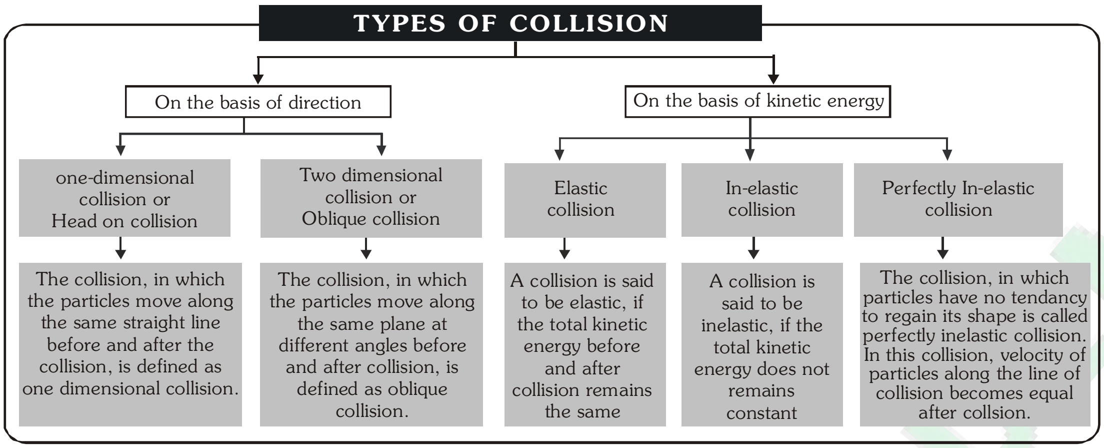
Force time graph area gives change in momentum.

Collision of bodies

The event or the process, in which two bodies either coming in contact with each other or due to mutual interaction at distance apart, affect each others motion (velocity, momentum, energy or direction of motion) is defined as a collision.

In collision

- The particles come closer before collision and after collision they either stick together or move away from each other.
- The particles need not come in contact with each other for a collision.
- The law of conservation of linear momentum is necessarily applicable in a collision, whereas the law of conservation of mechanical energy is not.

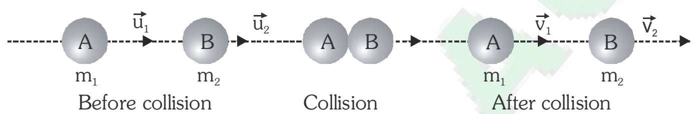


Coefficient of restitution (Newton's law)

$$e = -\frac{\text{velocity of separation along line of impact}}{\text{velocity of approach along line of impact}} = \frac{v_2 - v_1}{u_1 - u_2}$$

Value of e is 1 for elastic collision, 0 for perfectly inelastic collision and $0 < e < 1$ for inelastic collision.

Head on collision



Head on elastic collision

(i) Linear momentum is conserved

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

(ii) KE is not conserved but initial KE is equal to final KE

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

(iii) Rate of separation = Rate of approach

i.e. $e = 1$

$\Rightarrow u_1 - u_2 = v_2 - v_1$

Head on inelastic collision of two particles

Let the coefficient of restitution for collision is e

(i) Momentum is conserved $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ...(i) (ii) Kinetic energy is not conserved.

(iii) According to Newton's law $e = \frac{v_2 - v_1}{u_1 - u_2}$ ...(ii)

By solving eq. (i) and (ii) :

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2 = \frac{m_1u_1 + m_2u_2 - m_2e(u_1 - u_2)}{m_1 + m_2}$$

$$v_2 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 + \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1 = \frac{m_1u_1 + m_2u_2 - m_1e(u_2 - u_1)}{m_1 + m_2}$$

Elastic Collision (e=1)

- **If the two bodies are of equal masses :** $m_1 = m_2 = m$, $v_1 = u_2$ and $v_2 = u_1$

  Thus, if two bodies of equal masses undergo elastic collision in one dimension, then after the collision, the bodies will exchange their velocities.

- **If the mass of a body is negligible as compared to other.** If $m_1 >> m_2$ and $u_2 = 0$ then $v_1 = u_1$, $v_2 = 2u_1$

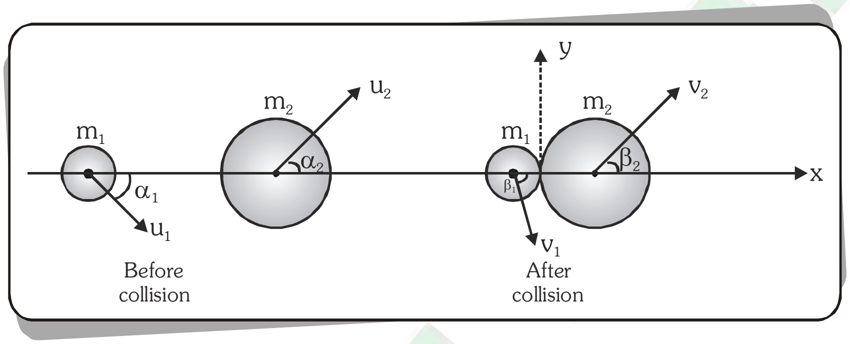
  when a heavy body A collides against a light body B at rest, the body A should keep on moving with same velocity and the body B will move with velocity double that of A. If $m_2 >> m_1$ and $u_2 = 0$ then $v_2 = 0$, $v_1 = -u_1$

  When light body A collides against a heavy body B at rest, the body A should start moving with same speed just in opposite direction while the body B should practically remains at rest.

- **Loss in kinetic energy in inelastic collision** $\Delta K = \frac{m_1 m_2}{2(m_1+m_2)}(1-e^2)\,|u_1-u_2|^2$

Oblique Collision

Conserving the momentum of system in directions along normal (x axis in our case) and tangential (y axis in our case) $m_1u_1\cos\alpha_1 + m_2u_2\cos\alpha_2 = m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2$



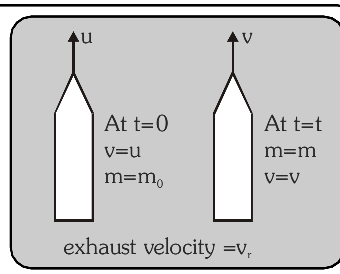
Since no force is acting on m_1 and m_2 along the tangent (i.e. y–axis) the individual momentum of m_1 and m_2 remains conserved. $m_1u_1\sin\alpha_1 = m_1v_1\sin\beta_1$ & $m_2u_2\sin\alpha_2 = m_2v_2\sin\beta_2$

By using Newton's experimental law along the line of impact $e = \frac{v_2\cos\beta_2 - v_1\cos\beta_1}{u_1\cos\alpha_1 - u_2\cos\alpha_2}$

Rocket propulsion :

Thrust force on the rocket= $v_r\left(-\frac{dm}{dt}\right)$

Velocity of rocket at any instant $v = u - gt + v_r\ell n\left(\frac{m_0}{m}\right)$



KEY POINTS

- Sum of mass moments about centre of mass is zero. i.e. $\sum m_i \vec{r}_{i/cm} = \vec{0}$
- A quick collision between two bodies is more violent then slow collision, even when initial and final velocities are equal because the rate of change of momentum determines that the impulsive force small or large.
- Heavy water is used as moderator in nuclear reactors as energy transfer is maximum if $m_1 \simeq m_2$
- Impulse-momentum theorem is equivalent to Newton's second law of motion.
- For a system, conservation of linear momentum is equivalent to Newton's third law of motion.