

Periodic Motion

Any motion which repeats itself after regular interval of time (i.e. time period) is called periodic motion or harmonic motion.

Example:

- (i) Motion of planets around the sun.
- (ii) Motion of the pendulum of wall clock.

Oscillatory Motion

The motion of body is said to be oscillatory or vibratory motion if it moves back and forth (to and fro) about a fixed point after regular interval of time.

The fixed point about which the body oscillates is called mean position or equilibrium position.

Example:

- (i) Vibration of the wire of 'Sitar'.
- (ii) Oscillation of the mass suspended from spring.

Simple Harmonic Motion (SHM)

Simple harmonic motion is the simplest form of vibratory or oscillatory motion. In which restoring force is directly proportional to distance from mean.

Some Basic Terms in SHM

- **Mean Position**

The point at which the restoring force on the particle is zero and potential energy is minimum, is known as its mean position.

- **Restoring Force**

The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force.

Restoring force always acts in a direction opposite to that of displacement. Displacement is measured from the mean position.

- **Amplitude**

The maximum (positive or negative) value of displacement of particle from mean position is defined as amplitude.

- **Time period (T)**

The minimum time after which the particle keeps on repeating its motion is known as time period.

The smallest time taken to complete one oscillation or vibration is also defined as time period.

It is given by $T = \frac{2\pi}{\omega} = \frac{1}{n}$ where ω is angular frequency and n is frequency.

- **Frequency (n or f)**

The number of oscillations per second is defined as

frequency. It is given by $n = \frac{1}{T} = \frac{\omega}{2\pi}$

- **Phase**

Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.

In the equation $x = A \sin(\omega t + \phi)$, $(\omega t + \phi)$ is the phase of the particle.

The phase angle at time $t = 0$ is known as initial phase or epoch.

The difference of total phase angles of two particles executing SHM with respect to the mean position is known as phase difference.

Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π , i.e. $\Delta\phi = 2n\pi$ where $n = 0, 1, 2, 3, \dots$

Two vibrating particle are said to be in opposite phase if the phase difference between them is an odd multiple of π i.e., $\Delta\phi = (2n + 1)\pi$ where $n = 0, 1, 2, 3, \dots$

- **Angular frequency (ω)** : The rate of change of phase angle of a particle with respect to time is defined as its angular frequency.

$$\omega = \sqrt{\frac{k}{m}}$$

- ♦ **For linear SHM**

$$(F \propto -x) : F = m \frac{d^2x}{dt^2} = -kx = -m\omega^2x \text{ where } \omega = \sqrt{\frac{k}{m}}$$

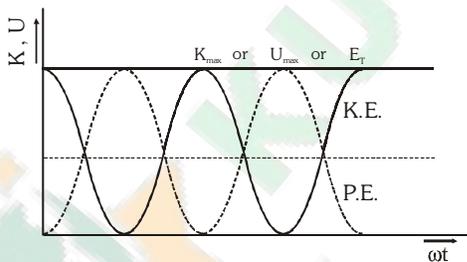
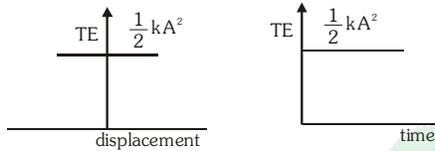
- ♦ **For angular SHM ($\tau \propto -\theta$) :**

$$\tau = I \frac{d^2\theta}{dt^2} = I\alpha = -k\theta = -m\omega^2\theta \text{ where } \omega = \sqrt{\frac{k}{m}}$$

- ♦ **Displacement** $x = A \sin(\omega t + \phi)$,

- ♦ **Angular displacement** $\theta = \theta_0 \sin(\omega t + \phi)$

- ♦ **Velocity** $v = \frac{dx}{dt} = A\omega \cos(\omega t + \phi) = \omega\sqrt{A^2 - x^2}$
- ♦ **Angular velocity** $\frac{d\theta}{dt} = \theta_0\omega \cos(\omega t + \phi)$
- ♦ **Acceleration** $a = \frac{d^2x}{dt^2} = -A\omega^2 \sin(\omega t + \phi) = -\omega^2 x$
- ♦ **Angular acceleration**
$$\frac{d^2\theta}{dt^2} = -\theta_0\omega^2 \sin(\omega t + \phi) = -\omega^2 \theta$$
- ♦ **Kinetic energy** $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi)$
- ♦ **Potential energy** $U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$
- ♦ **Total energy** $E = K + U = \frac{1}{2}m\omega^2 A^2 = \text{constant}$



Note :

- (i) Total energy of a particle in S.H.M. is same at all instant and at all displacement.
- (ii) Total energy depends upon mass, amplitude and frequency of vibration of the particle executing S.H.M.

Average energy in SHM

(i) The time average of P.E. and K.E. over one cycle is

$$(a) \langle K \rangle_t = \frac{1}{4}kA^2 \quad (b) \langle PE \rangle_t = \frac{1}{4}kA^2$$

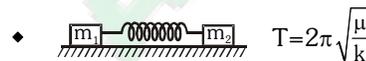
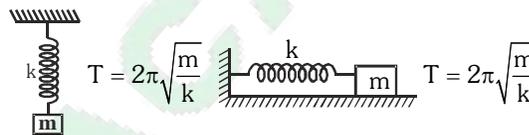
$$(c) \langle TE \rangle_t = \frac{1}{2}kA^2 + U_0$$

(ii) The position average of P.E. and K.E. between $x = -A$ to $x=A$

$$(a) \langle K \rangle_x = \frac{1}{3}kA^2 \quad (b) \langle PE \rangle_x = U_0 + \frac{1}{6}kA^2$$

$$(c) \langle TE \rangle_x = \frac{1}{2}kA^2 + U_0$$

♦ **Spring block system**

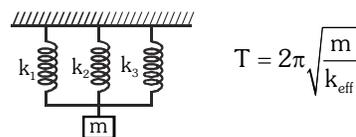


where $\mu = \text{reduced mass} = \frac{m_1 m_2}{m_1 + m_2}$

♦ **Series combination of springs**

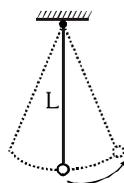
$$T = 2\pi \sqrt{\frac{m}{k_{\text{eff}}}} \quad \text{where} \quad \frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

♦ **Parallel combination of springs**



where $k_{\text{eff}} = k_1 + k_2 + k_3$

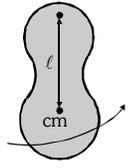
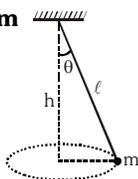
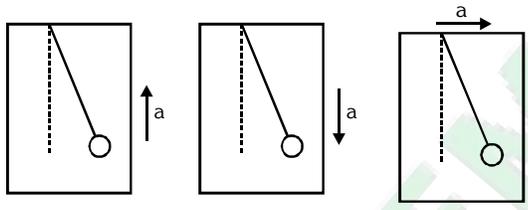
♦ **Time period of simple pendulum**



$$\text{Time period } T = 2\pi \sqrt{\frac{L}{g}}$$

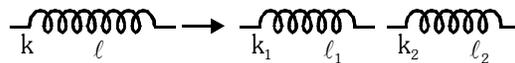
♦ **Second pendulum**

Time period = 2 seconds, Length \approx 1 meter (on earth's surface)

<ul style="list-style-type: none"> ◆ Time period of Physical pendulum $T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{k^2 + \ell}{g}}$ <p>where $I_{cm} = mk^2$</p>  <ul style="list-style-type: none"> ◆ Time period of Conical pendulum $T = 2\pi\sqrt{\frac{\ell \cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}}$  <ul style="list-style-type: none"> ◆ Time period of Torsional pendulum $T = 2\pi\sqrt{\frac{I}{k}}$ <p>where k = torsional constant of the wire I = moment of inertia of the body about the vertical axis</p>	<p>In accelerating cage</p>  <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;">$g_{\text{eff}} = g + a$</div> <div style="text-align: center;">$g_{\text{eff}} = g - a$</div> <div style="text-align: center;">$g_{\text{eff}} = \sqrt{g^2 + a^2}$</div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;">$T = 2\pi\sqrt{\frac{\ell}{g+a}}$</div> <div style="text-align: center;">$T = 2\pi\sqrt{\frac{\ell}{g-a}}$</div> <div style="text-align: center;">$T = 2\pi\sqrt{\frac{\ell}{(g^2 + a^2)^{1/2}}}$</div> </div>
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KEY POINTS

- SHM is the projection of uniform circular motion along one of the diameters of the circle.
- The periodic time of a hard spring is less as compared to that of a soft spring because the spring constant is large for hard spring.
- For a system executing SHM, the mechanical energy remains constant.
- Maximum kinetic energy of a particle in SHM may be greater than mechanical energy as potential energy of a system may be negative.
- The frequency of oscillation of potential energy and kinetic energy is twice as that of displacement or velocity or acceleration of a particle executing S.H.M.
- ◆ **Spring cut into two parts :**



Here $\frac{l_1}{l_2} = \frac{m}{n}$ $l_1 = \left(\frac{m}{m+n}\right)\ell$, $l_2 = \left(\frac{n}{m+n}\right)\ell$ But $k\ell = k_1l_1 = k_2l_2 \Rightarrow k_1 = \frac{(m+n)}{m}k$; $k_2 = \frac{(m+n)}{n}k$