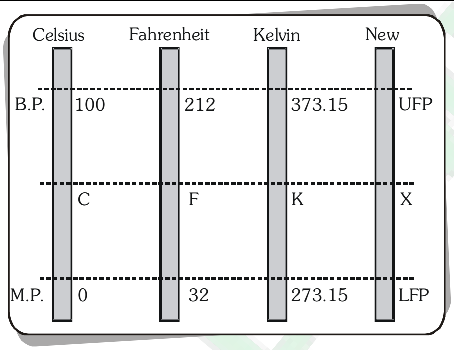
THERMAL PHYSICS

TEMPERATURE SCALES AND THERMAL EXPANSION

Name of the scale	Symbol for each degree	Lower fixed point (LFP)	Upper fixed point (UFP)	Number of divisions on the scale
Celsius	°C	0°C	100°C	100
Fahrenheit	°F	32°F	212°F	180
Kelvin	K	273.15 K	373.15 K	100

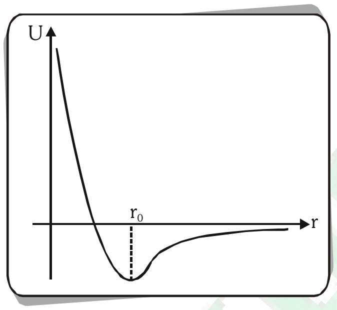


$$\frac{C-0}{100-0}=\frac{F-32}{212-32}=\frac{K-273.15}{373.15-273.15}=\frac{X-LFP}{UFP-LFP}\Rightarrow\frac{\Delta C}{100}=\frac{\Delta F}{180}=\frac{\Delta K}{100}=\frac{\Delta X}{UFP-LFP}$$

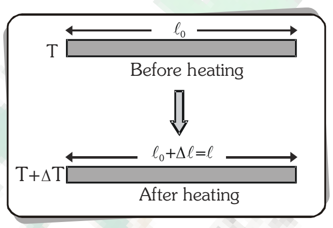
- **Old thermometry** : $\frac{\theta-0}{100-0}=\frac{X-X_0}{X_{100}-X_0}$ [two fixed points –ice & steam points] where X is thermometric property i.e. length, resistance etc.
- **Modern thermometry** : $\frac{T-0}{273.16-0}=\frac{X}{X_{tr}}$ [Only one reference point–triple point of water is chosen]

THERMAL EXPANSION

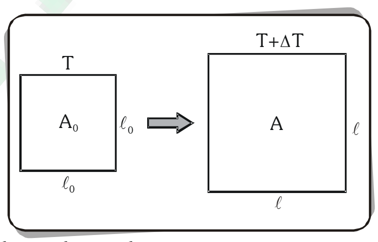
It is due to asymmetry in potential energy curve.



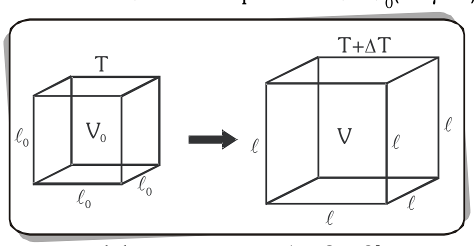
In solids → Linear expansion $\ell=\ell_0(1+\alpha\Delta T)$



In solids → Areal expansion $A=A_0(1+\beta\Delta T)$



In solids, liquids and gases →
Volume expansion $V=V_0(1+\gamma\Delta T)$



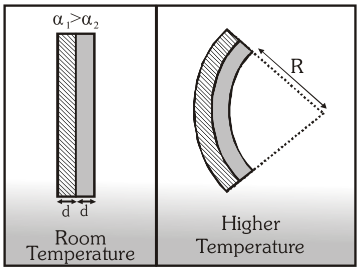
[For isotropic solids : $\alpha:\beta:\gamma=1:2:3$]
Thermal expansion of an isotropic object may be imagined as a photographic enlargement.
For anisotropic materials $\beta_{xy}=\alpha_x+\alpha_y$ and $\gamma=\alpha_x+\alpha_y+\alpha_z$

If α is variable : $\Delta\ell=\int_{T_1}^{T_2}\ell_0\alpha dT$

Measurement of length by metallic scale
Measured value at temp θ_2 °C,
$MV=\ell_a\{1+(\alpha_0-\alpha_s)(\theta_2-\theta_1)\}$
where,
ℓ_a = actual length of object at θ_1 °C
α_0 = linear expansion coefficient of object.
α_s = linear expansion coefficient of scale.
(i) if $\alpha_0>\alpha_s$, then MV is more than ℓ_a
(ii) if $\alpha_0<\alpha_s$, then MV is less than ℓ_a

Application of Thermal expansion in solids

I. Bi-metallic strip *(used as thermostat or auto-cut in electric heating circuits)* $R = \dfrac{d}{(\alpha_1 - \alpha_2)\Delta T}$



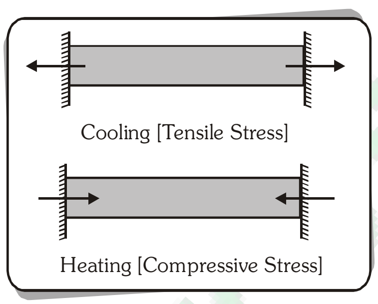
II. Simple pendulum :

$$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T \propto \ell^{1/2} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$$

Fractional change in time period $= \dfrac{\Delta T}{T} = \dfrac{1}{2}\alpha\Delta\theta$

III. Scale reading : Due to linear expansion / contraction, scale reading will be lesser / more than actual value.
If temperature ↑ then actual value = scale reading $(1+\alpha\Delta\theta)$

IV. Thermal Stress



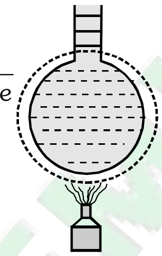
Thermal strain $= \dfrac{\Delta \ell}{\ell} = \alpha\Delta\theta$

As Young's modulus $Y = \dfrac{F/A}{\Delta\ell/\ell}$;

So thermal stress $= Y\alpha\Delta\theta = \dfrac{YA\alpha\Delta\theta}{(1+\alpha\Delta\theta)}$

Thermal expansion in liquids

(Only volume expansion)



$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Initial volume} \times \text{Temperature rise}}$$

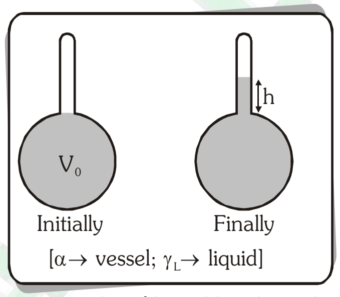
$$\gamma_r = \frac{\text{real increse in volume}}{\text{initial volume} \times \text{temperature rise}}$$

$$\gamma_r = \gamma_a + \gamma_{vessel}$$

Change in volume of liquid w.r.t. vessel

$$\Delta V = V_0(\gamma_r - 3\alpha)\Delta T$$

Expansion in enclosed volume



Increase in height of liquid level in tube when bulb was initially completely filled.

$$h = \frac{\text{apparent change in volume of liquid}}{\text{area of tube}} = \frac{V_0(\gamma_L - 3\alpha)\Delta T}{A_0(1+2\alpha)\Delta T}$$

Anomalous expansion of water :
In the range 0°C to 4 °C water contract on heating and expands on cooling. At 4°C → density is maximum.
Aquatic life is able to survive in very cold countries as the lake bottom remains unfrozen at the temperature around 4°C.

Thermal expansion of gases :

- Coefficient of volume expansion : $\gamma_V = \dfrac{\Delta V}{V_0 \Delta T} = \dfrac{1}{T}$

  [PV = nRT at constant pressure $V \propto T \Rightarrow \dfrac{\Delta V}{V} = \dfrac{\Delta T}{T}$]

- Coefficient of pressure expansion $\gamma_P = \dfrac{\Delta P}{P_0 \Delta T} = \dfrac{1}{T}$

KEY POINTS :

- Liquids usually expand more than solids because the intermolecular forces in liquids are weaker than in solids.
- Rubber contract on heating because in rubber as temperature increases, the amplitude of transverse vibrations increases more than the amplitude of longitudinal vibrations.
- Water expands both when heated or cooled from 4°C because volume of water at 4°C is minimum.
- In cold countries, water pipes sometimes burst, because water expands on freezing.

CALORIMETRY

1 cal = 4.186 J ≃ 4.2 J

- **Thermal capacity of a body** $= \frac{Q}{\Delta T}$

  Amount of heat required to raise the temperature of a given body by 1°C (or 1K).

- **Specific heat capacity** $= \frac{Q}{m\Delta T}$ (m = mass)

  Amount of heat required to raise the temperature of unit mass of a body through 1°C (or 1K)

- **Molar heat capacity** $= \frac{Q}{n\Delta T}$ (n=number of moles)

- **Water equivalent :** If thermal capacity of a body is expressed in terms of mass of water, it is called water equivalent. Water equivalent of a body is the mass of water which when given same amount of heat as to the body, changes the temperature of water through same range as that of the body. Therefore water equivalent of a body is the quantity of water, whose heat capacity is the same as the heat capacity of the body.

  Water equivalent of the body,

  $$W = \text{mass of body} \times \left(\frac{\text{specific heat of body}}{\text{specific heat of water}}\right)$$

  Unit of water equivalent is g or kg.

- **Latent Heat (Hidden heat) :** The amount of heat that has to supplied to (or removed from) a body for its complete change of state (from solid to liquid, liquid to gas etc) is called latent heat of the body. Remember that phase transformation is an isothermal (i.e. temperature = constant) change.

- **Principle of calorimetry :**

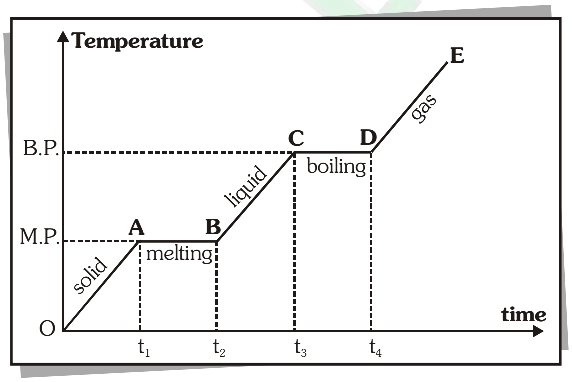
  Heat lost = heat gained

  For temperature change $Q = ms\Delta T$,

  For phase change $Q = mL$

- **Heating curve :**

  If to a given mass (m) of a solid, heat is supplied at constant rate (Q) and a graph is plotted between temperature and time, the graph is called heating curve.



Specific heat $\propto \frac{1}{\text{slope of curve}}$

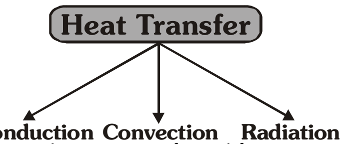
(or thermal capacity)

Latent heat ∝ length of horizontal line.

KEY POINTS

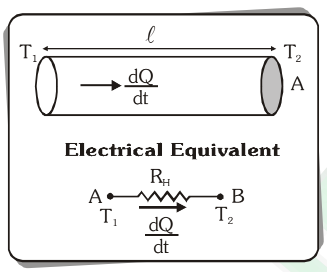
- Specific heat of a body may be greater than its thermal capacity as mass of the body may be less than unity.
- The steam at 100°C causes more severe burn to human body than the water at 100°C because steam has greater internal energy than water due to latent heat of vaporization.
- Heat is energy in transit which is transferred from hot body to cold body.
- One calorie is the amount of heat required to raise the temperature of one gram of water through 1°C (more precisely from 14.5°C to 15.5°C).
- Clausins & clapeyron equation (effect of pressure on boiling point of liquids & melting point of solids related with latent heat) $\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$

THERMAL CONDUCTION



In conduction, heat is transferred from one point to another without the actual motion of heated particles. In the process of convection, the heated particles of matter actually move. In radiation, intervening medium is not affected and heat is transferred without any material medium.

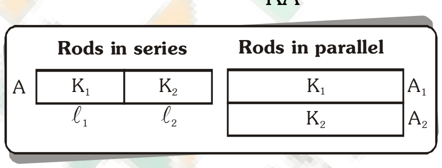
Conduction	*Convection*	*Radiation*
Heat Transfer due to Temperature difference	Heat transfer due to density difference	Heat transfer with out any medium
Due to free electron or vibration motion of molecules	Actual motion of particles	Electromagnetic radiation
Heat transfer in solid body (in mercury also)	Heat transfer in fluids (Liquid + gas)	All
Slow process	Slow process	Fast process (3×10^8 m/sec)
Irregular path	Irregular path	Straight line (like light)



Rate of heat flow

$$\frac{dQ}{dt} = -KA\frac{dT}{dx} \quad \text{or} \quad \frac{Q}{t} = \frac{KA(T_1 - T_2)}{\ell}$$

Thermal resistance $R_H = \frac{\ell}{KA}$

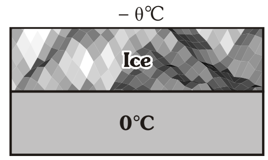


$$K_{eq} = \frac{\Sigma \ell}{\Sigma \ell / K} \;;\; K_{eq} = \frac{\Sigma KA}{\Sigma A}$$

Growth of Ice on Ponds

Time taken by ice to grow a thickness from x_1 to x_2: $t = \frac{\rho L}{2K\theta}(x_2^2 - x_1^2)$

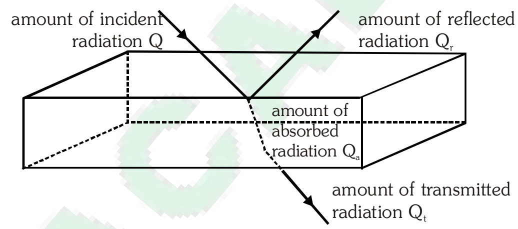
[K=thermal conductivity of ice, ρ=density of ice]



RADIATION

- **Spectral, emissive, absorptive and transmittive power of a given body surface:** Due to incident radiations on the surface of a body following phenomena occur by which the radiation is divided into three parts.

  (a) Reflection (b) Absorption (c) Transmission



From energy conservation

$$Q = Q_r + Q_a + Q_t \Rightarrow \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q} = 1$$

$$\Rightarrow r + a + t = 1$$

- Reflective Coefficient : $r = \frac{Q_r}{Q}$
- Absorptive Coefficient : $a = \frac{Q_a}{Q}$
- Transmittive Coefficient : $t = \frac{Q_t}{Q}$

  r = 1 and a = 0 , t = 0 ⇒ Perfect reflector

  a = 1 and r = 0, t = 0 ⇒ Ideal absorber (ideal black body)

  t = 1 and a = 0, r = 0 ⇒ Perfect transmitter (daithermanous)

  Reflection power (r) = $\left[\frac{Q_r}{Q} \times 100\right]\%$

  Absorption power (a) = $\left[\frac{Q_a}{Q} \times 100\right]\%$

  Transmission power (t) = $\left[\frac{Q_t}{Q} \times 100\right]\%$

- **Stefan's Boltzmann law :** Radiated energy emitted by a perfect black body per unit area/sec $E = \sigma T^4$

  For a general body $E = \sigma e_r T^4$ [where $0 \le e_r \le 1$]

- **Prevost's theory of heat exchange :** A body is simultaneously emitting radiations to its surrounding and absorbing radiations from the surroundings. If surrounding has temperature T_0 then $E_{net} = e_r\sigma(T^4 - T_0^4)$

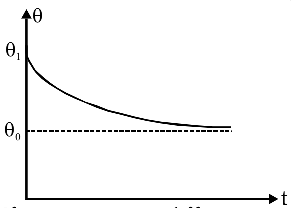
- **Kirchhoff's law :**
  The ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.
  $$\frac{e}{a}=\frac{E}{A}=\frac{E}{1}\Rightarrow\frac{e}{a}=E\Rightarrow e\propto a$$
  Therefore a good absorber is a good emitter.

- **Perfectly Black Body :**
  A body which absorbs all the radiations incident on it is called a perfectly black body.

- **Absorptive Power (a) :**
  Absorptive power of a surface is defined as the ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it in the same time.
  For ideal black body, absorptive power =1

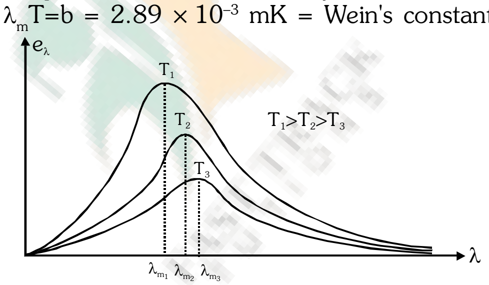
- **Emissive power(e) :**
  For a given surface it is defined as the radiant energy emitted per second per unit area of the surface.

- **Newton's law of cooling:**



  If temperature difference is small
  Rate of cooling
  $$\frac{d\theta}{dt}\propto(\theta-\theta_0)\Rightarrow\theta=\theta_0+(\theta_1-\theta_0)e^{-kt}$$
  [where k = constant]
  when a body cools from θ_1 to θ_2 in time 't' in a surrounding of temperature θ_0 then
  $$\frac{\theta_1-\theta_2}{t}=k\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$$ [where k= constant]

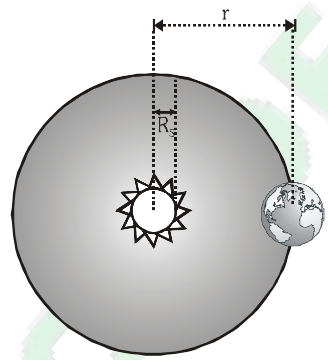
- **Wien's Displacement Law :**
  Product of the wavelength λ_m of most intense radiation emitted by a black body and absolute temperature of the black body is a constant
  $\lambda_m T=b = 2.89\times10^{-3}$ mK = Wein's constant



Area under $e_\lambda-\lambda$ graph$=\int_0^\infty e_\lambda d\lambda=e=\sigma T^4$

Solar constant

The Sun emits radiant energy continuously in space of which an insignificant part reaches the Earth. The solar radiant energy received per unit area per unit time by a black surface held at right angles to the Sun's rays and placed at the mean distance of the Earth (in the absence of atmosphere) is called solar constant.



$$S=\frac{P}{4\pi r^2}=\frac{4\pi R_S^2\sigma T^4}{4\pi r^2}=\sigma\left(\frac{R_S}{r}\right)^2T^4$$

where R_S = radius of sun
r = average distance between sun and earth.

Note :- S = 2 cal $cm^{-2}min$=1.4 kWm^{-2}
T = temperature of sun ≈ 5800 K

KEY POINTS

- Stainless steel cooking pans are preferred with extra copper bottom because thermal conductivity of copper is more than steel.
- Two layers of cloth of same thickness provide warmer covering than a single layer of cloth of double the thickness because air (which is better insulator of heat) is trapped between them.
- Animals curl into a ball when they feel very cold to reduce the surface area of the body.
- Water cannot be boiled inside a satellite by convection because in weightlessness conditions, natural movement of heated fluid is not possible.
- Metals have high thermal conductivity because metals have free electrons.