KUSAL ACADEMY

ELECTROSTATICS

ELECTRIC CHARGE

Charge of a material body is that property due to which it interacts with other charges. There are two kinds of charges- positive and negative.

S.I. unit → Coulomb (C)

Properties of charge :-

(a) Charge is a scalar quantity (b) Charge is quantised

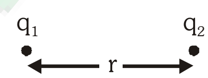
(c) Charge is conserved. (d) Charge is independent of frame of reference.

Methods of charging :-

(a) Friction (b) Induction (c) Conduction

COULOMB'S LAW

Force between two charges $\vec{F} = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$



where, $\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

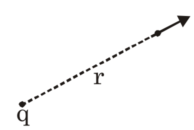
If medium is present then $\vec{F} = \frac{1}{4\pi \epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2} \hat{r}$

NOTE : The Law is applicable only for static and point charges. Moving charges may result in magnetic interaction. And if charges are spread on bodies then induction may change the charge distribution.

ELECTRIC FIELD OR ELECTRIC INTENSITY OR ELECTRIC FIELD STRENGTH

Electric field intensity is defined as force on unit test charge.

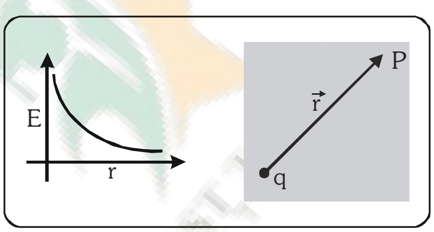
$\vec{E} = \frac{kq}{r^2}\hat{r} = \frac{kq}{r^3}\vec{r}$



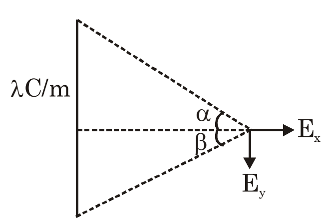
SI unit : Newton/coulomb (N/C)

ELECTRIC FIELD DUE TO SPECIAL CHARGE DISTRIBUTION

(a) Due to point charge $\vec{E} = \frac{kq}{r^2}\hat{r}$



(b) Due to linear change distribution :-

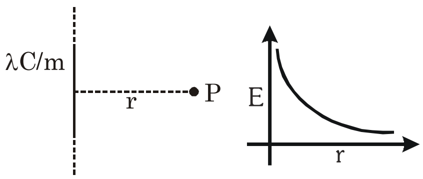


$E_x = \frac{\lambda}{4\pi \epsilon_0 r}(\sin\alpha + \sin\beta)$

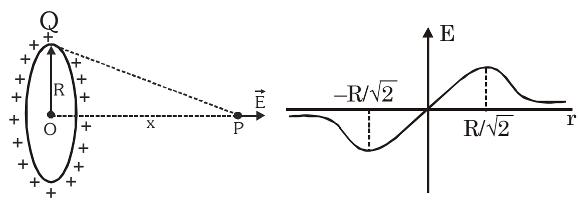
$E_y = \frac{\lambda}{4\pi \epsilon_0 r}(\cos\beta - \cos\alpha)$

(c) Due to infinite line of charge

$$\vec{E}_P = \frac{\lambda}{2\pi \epsilon_0 r}\hat{r}$$



(d) Electric field due to uniformly changed ring

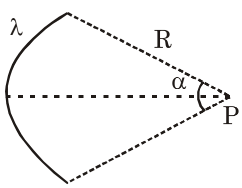


$$E_P = \frac{kQx}{\left(R^2 + x^2\right)^{3/2}}$$

(i) At centre of the ring, x = 0. So E = 0

(ii) Electric field is maximum at $x = \pm \frac{R}{\sqrt{2}}$

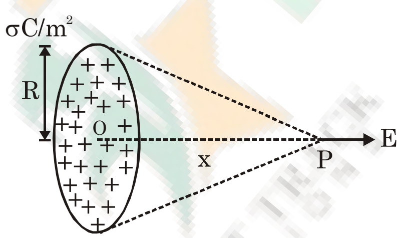
(e) Due to segment of ring



$$E_P = \frac{2k\lambda}{R}\sin\left(\frac{\alpha}{2}\right)$$

Direction of electric field is along the direction of angle bisector of the arc.

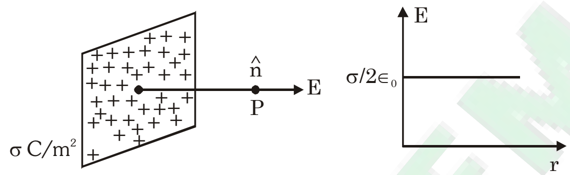
(f) Due to charged disk



$$E_P = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

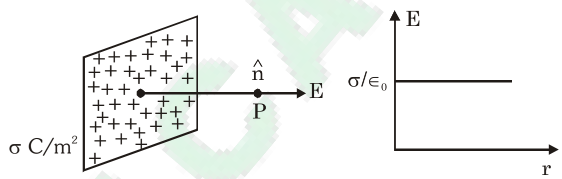
(g) Due to infinite plane sheet of charge

$$\vec{E}_P = \frac{\sigma}{2\epsilon_0}\hat{n}$$

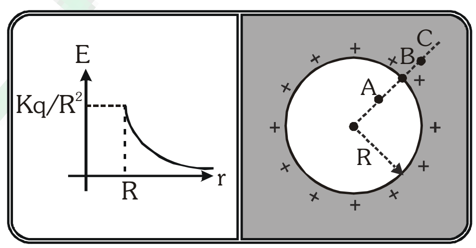


(h) Due to infinite charged conducting plate

$$\vec{E}_P = \frac{\sigma}{\epsilon_0}\hat{n}$$



(i) Due to hollow non-conducting sphere

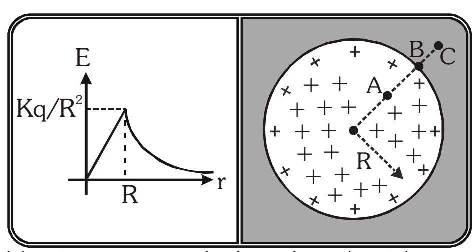


(a) For point inside the sphere (r < R) : $E_A = 0$

(b) For point on the surface (r = R) : $E_B = \frac{kQ}{R^2}$

(c) For point outside the sphere : $E_C = \frac{kQ}{r^2}$

(j) Due to uniformly charged non-conducting sphere



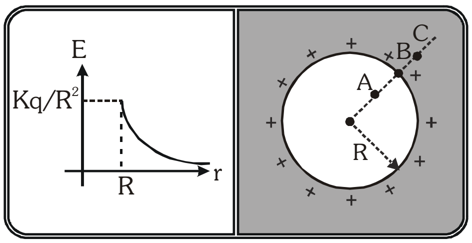
(a) For point inside the sphere (r < R)

$$E_A = \frac{kQr}{R^3} = \frac{\rho r}{3\epsilon_0}$$

(b) For point on the surface (r = R) : $E_B = \dfrac{kQ}{R^2}$

(c) For point outside the sphere (r > R) : $E_C = \dfrac{kQ}{r^2}$

(k) Due to solid or Hollow conducting sphere

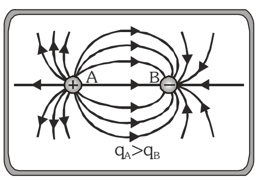


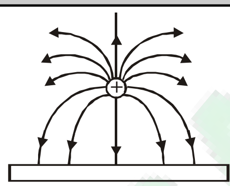
(a) For point inside the sphere (r < R) : $E_A = 0$

(b) For point on the surface (r = R): $E_B = \dfrac{kQ}{R^2}$

(c) For point outside the sphere (r > R) : $E_C = \dfrac{kQ}{r^2}$

ELECTRIC FIELD LINES





Fixed point charge near infinite metal plate

Electric field lines have the following properties :-

(a) Imaginary curves

(b) Never intersect each other

(c) Never form closed loops

(d) Start from (+ve) charge and ends on (–ve) charge.

(e) If there is no electric field then there will no field lines

(f) Number of electric field lines per unit area normal to the area at a point represents magnitude of electric field intensity. Crowded lines represent strong field while distant lines weak field.

(g) Number of lines originating from or terminating on a charge is proportional to magnitude of charge.

(h) Field lines start or end normally at the surface of a conductor.

(i) Tangent to the lines of force at a point in an electric field gives direction of intensity of electric field.

ELECTRIC FLUX

$$\phi = \int \vec{E}.\vec{dA}$$

(a) Scalar quantity

(b) SI unit :- Nm^2/C or V–m

(i) For uniform electric field $\phi = \vec{E}.\vec{A} = EA\cos\theta$

where, θ = Angle between $\vec{E}$ and area vector ($\vec{A}$).

(ii) For non-uniform field $\phi = \int \vec{E}.\vec{dA}$

Gauss's Law

For a closed surface, total flux $\phi = \oint \vec{E}.\vec{dA} = \dfrac{q_{in}}{\epsilon_0}$

where q_{in} = net charge enclosed by the closed surface.

(i) Flux through Gaussian surface is independent of its shape.

(ii) Flux depends only on charges present inside the closed surface.

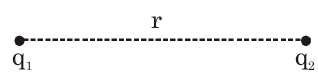
(iii) Flux through a closed surface is independent of position of charges inside it.

(iv) Electric field intensity at the Gaussian surface is due to all charges present (inside as well as outside).

ELECTROSTATIC POTENTIAL ENERGY

It is the amount of energy required to bring any charge from ∞ to any particular point without any charge in KE.

Interection energy of a system of two charged particles



$$U = \frac{kq_1q_2}{r}$$

{ Assuming potential energy at ∞ to be zero}

ELECTRIC POTENTIAL

It is the work done against the field to take a unit positive charge from infinity (reference point) to the given point P without gaining any kinetic energy.

$$V_P = \frac{(W_{\infty-p})_{ext}}{q} = \frac{U}{q}$$

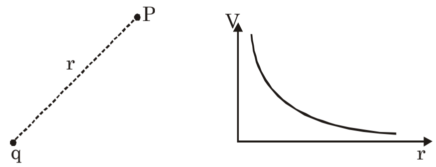
(i) Electric potential is a scalar quantity

(ii) SI unit :- Volt (V) or J/C

(iii) In presence of dielectric medium, potential decreases and becomes $\dfrac{1}{\varepsilon_r}$ times of its free space value.

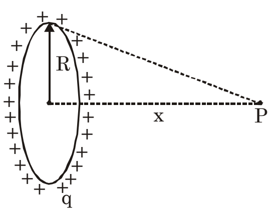
ELECTRIC POTENTIAL DUE TO SPECIAL CHARGE DISTRIBUTION :-

(a) Due to a point charge :-



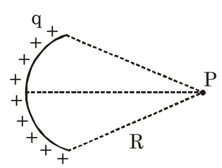
$$V_P = \frac{kq}{r} = \frac{q}{4\pi\varepsilon_0 r}$$

(b) Due to a charged ring :-



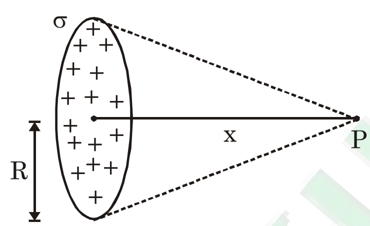
$$V_P = \frac{kq}{\sqrt{R^2 + x^2}}$$

(c) Due to segment of ring :-



$$V_P = \frac{kQ}{R}$$

(d) due to charged disk :-

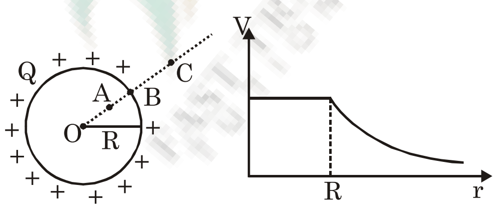


$$V_P = \frac{\sigma}{2\epsilon_0}\left(\sqrt{x^2 + R^2} - x\right)$$

(e) Due to non-conducting spherical shell :-

(a) For point inside the sphere (r < R) :-

$$V_A = \frac{kQ}{R}$$



(b) For point on the surface (r = R) :-

$$V_B = \frac{kQ}{R}$$

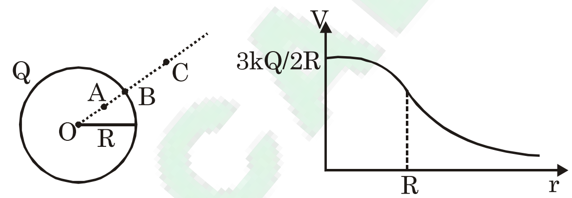
(c) For point outside the sphere (r > R) :-

$$V_C = \frac{kQ}{r}$$

(f) Due to solid non-conducting sphere :-

(a) For point inside the sphere (r < R) :-

$$V_A = \frac{kQ}{2R^3}\left(3R^2 - r^2\right)$$



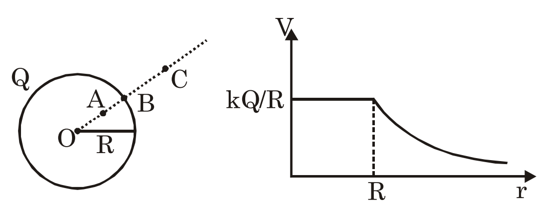
(b) For point on the surface (r = R)

$$V_B = \frac{kQ}{R}$$

(c) For point outside the sphere (r > R) :-

$$V_C = \frac{kQ}{r}$$

(g) Due to conducting sphere or shell :-



(a) For point inside the sphere (r < R) :-

$$V_A = \frac{kQ}{R}$$

(b) For point on the surface (r = R) :-

$$V_B = \frac{kQ}{R}$$

(c) For point outside the surface (r > R) :-

$$V_C = \frac{kQ}{r}$$

POTENTIAL DIFFERENCE

The potential difference between two points A & B is work done by external agent against electric field in taking a unit positive charge from B to A keeping kinetic energy constant :

$$V_A - V_B = \frac{(W_{BA})_{ext}}{q}$$

Relation between electric field & electric potential :-

$$\vec{E} = -\vec{\Delta}V = -\text{grad}V = -\frac{dV}{dr}\hat{r}$$

$$\vec{E} = \frac{-\partial v}{\partial x}\hat{i} - \frac{\partial v}{\partial y}\hat{j} - \frac{\partial v}{\partial z}\hat{k} \;;\; V = -\int \vec{E}.\vec{dr}$$

(a) Direction of $\vec{E}$ is from high potential to low potential.

(b) If V = constant over a region, then E = 0 (in that region)

EQUIPOTENTIAL SURFACE.

The locus of all points having same potential is called

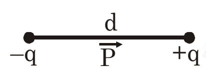
(a) Equipotential surfaces can never cross each other.

(b) Equipotential surfaces are always perpendicular to the direction of electric field.

(c) No work is done in moving a charge from one point to other over an equipotential surface.

ELECTRIC DIPOLE

A system of two equal and opposite charges separated by a small distance is called electric dipole :



Dipole Moment :- $\vec{p} = q\vec{d}$

Direction of dipole moment is from negative to positive charge :

DIPOLE PLACED IN UNIFORM ELECTRIC FIELD

(a) Torque $\vec{\tau} = \vec{p} \times \vec{E}$

(b) Net force = 0

(c) Work done in rotation of dipole from θ_1 to θ_2 angle in external electric field $W = pE(\cos\theta_1 - \cos\theta_2)$

(d) Electrostatic potential energy = $-\vec{p}.\vec{E} = -pE\cos\theta$

(e) In non-uniform electric field, force on electric dipole

$$\vec{F} = -\vec{p}.\frac{d\vec{E}}{dr}$$

ELECTRIC FIELD DUE TO DIPOLE

(a) At an axial point :-

$$\vec{E} = \frac{2k\vec{p}}{r^3}$$

(b) On the equitorial line :-

$$\vec{E} = \frac{-k\vec{p}}{r^3}$$

(c) At any general point :-

$$\vec{E} = \frac{k\vec{p}}{r^3}\sqrt{1+3\cos^2\theta}$$

ELECTRIC POTENTIAL DUE TO DIPOLE

(a) At an axial point :-

$$V = \frac{kp}{r^2}$$

(b) At equitorial point :-

$V = 0$

(c) At a general point :-

$$V = \frac{kp\cos\theta}{r^2}$$

CONDUCTORS AND ITS PROPERTIES

(a) Conductors are always equipotential surfaces.

(b) Charge always reside on the outer surface of a conductor.

(c) Electric field is always perpendicular to conducting surface.

(d) Electric field lines never exist within conducting materials.

(e) When a conductor is grounded, its potential becomes zero.