GRAVITATION

Newton's law of gravitation

$m_1 \longleftarrow r \longrightarrow m_2$

Force of attraction between two point masses

$$F = \frac{Gm_1m_2}{r^2}$$

Directed along the line joining of point masses.

- It is a conservative force field ⇒ mechanical energy is conserved.
- It is a central force field ⇒ angular momentum is conserved.

$$G = 6.67 \times 10^{-11} \frac{m^3}{kgs^2}$$

Gravitational field due to point mass at distance x

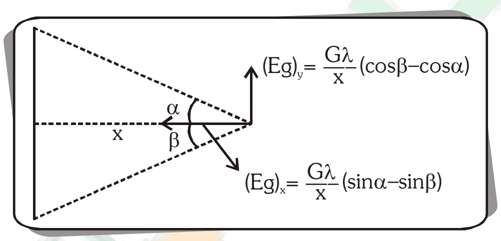
$$Eg = \frac{Gm}{r^2} \quad \text{[Radially inwards]}$$

Gravitational field on the axis of uniform thin ring at distance x

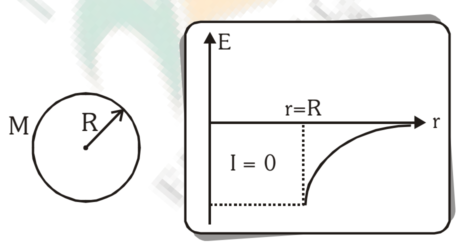
$$Eg = \frac{Gm}{(R^2 + x^2)^{3/2}} x \quad \text{[Directed towards centre]}$$

Eg is max at $x = \pm \frac{R}{\sqrt{2}}$

Uniform linear mass (mass density λ)



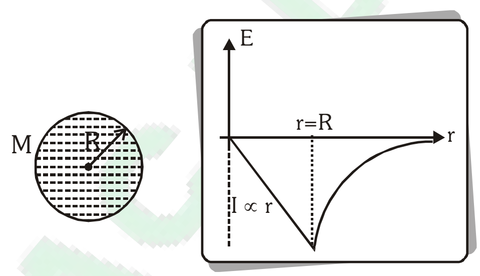
Gravitational field due to spherical shell



- Outside the shell $E_g = \frac{GM}{r^2}$, where r > R
- On the surface $E_g = \frac{GM}{r^2}$, where r=R
- Inside the shell $E_g = 0$, where r<R

[Note:Direction always towards the centre of the sphere]

Gravitational field due to solid sphere



- Outside the sphere $E_g = \frac{GM}{r^2}$, where r > R
- On the surface $E_g = \frac{GM}{r^2}$, where r=R
- Inside the sphere $E_g = \frac{GMr}{R^3}$, where r< R

Acceleration due to gravity $g = \frac{GM}{R^2}$

- At height h : $g_h = \frac{GM}{(R+h)^2}$

  If h << R : $g_h \approx g_s\left(1 - \frac{2h}{R}\right)$

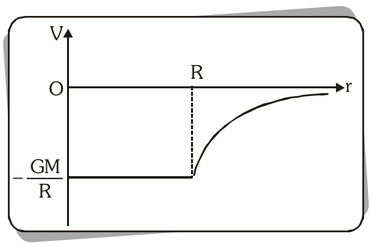
- At depth d : $g_d = \frac{GM(R-d)}{R^3} = g_s\left(1 - \frac{d}{R}\right)$
- Effect of rotation on g : $g' = g - \omega^2 R\cos^2\lambda$

  where λ is angle of latitude.

Gravitational potential

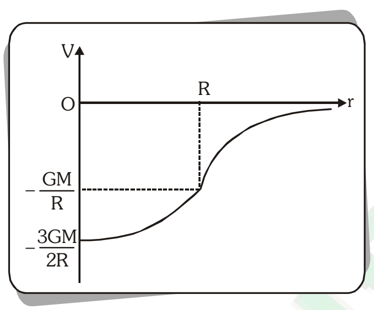
Due to a point mass at a distance $V = -\frac{GM}{r}$

Gravitational potential due to spherical shell



- Outside the shell $V = -\frac{GM}{r}$, $r>R$
- Inside/on the surface the shell $V = -\frac{GM}{R}$, $r \le R$

Potential due to solid sphere



- Outside the sphere $V = -\frac{GM}{r}$, $r > R$
- On the surface $V = -\frac{GM}{R}$, $r = R$
- Inside the sphere $V = -\frac{GM(3R^2 - r^2)}{2R^3}$, $r < R$

Potential on the axis of a thin ring at a distance x

$$V = -\frac{GM}{\sqrt{R^2 + x^2}}$$

Electrostatic self-energy

- For two point masses $U = -\frac{Gm_1m_2}{r}$
- Uniform thin spherical shell $U = -\frac{GM^2}{2R}$
- Uniform solid sphere $U = \frac{3}{2}\frac{GM^2}{R}$

Escape velocity from the surface a planet of mass M & radius R

$$v_e = \sqrt{\frac{2GM}{R}}$$

Orbital velocity of satellite

$$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$$

- For nearby satellite $v_0 = \sqrt{\frac{GM}{R}} = \frac{v_e}{\sqrt{2}}$

  Here v_e = escape velocity on earth surface.

Time period of satellite

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

Energies of a satellite

- Potential energy $U = -\frac{GMm}{r}$
- Kinetic energy $K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$
- Mechanical energy $E = U + K = -\frac{GMm}{2r}$
- Binding energy $BE = -E = \frac{GMm}{2r}$

Kepler's laws

- **I^st^** Law (Law of orbit) Path of a planet is elliptical with the sun at a focus.
- **II^nd^** Law (Law of area)

  Areal velocity $\frac{dA}{dt}$ = constant $= \frac{L}{2m}$
- **III^rd^** Law (Law of period) $T^2 \propto a^3$ or

$$T^2 \propto \left(\frac{r_{max} + r_{min}}{2}\right)^3 \propto (\text{mean radius})^3$$

For circular orbits $T^2 \propto R^3$