KUSAL ACADEMY

CURRENT ELECTRICTY

Some Definations

ELECTRIC CURRENT

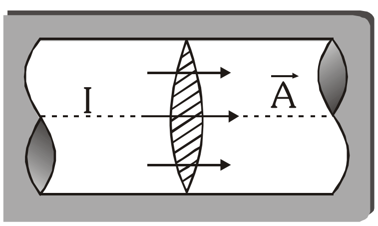
The rate of flow of electric charge across any cross-section is called electric current.

(a) Instantaneous electric current $I = \frac{dq}{dt}$

(b) Average electric current $I_{av} = \frac{\Delta q}{\Delta t}$

Current Density

Current flowing per unit area through any cross-section is called current density.



$J = \frac{I}{A}$

$$I = \vec{J}.\vec{A} = JA\cos\theta$$

Drift Velocity

Average velocity with which electrons drift from low potential end to high potential end of the conductor (v_d). Drift velocity is given by

$\vec{v}_d = -\frac{e\tau}{m}\vec{E}$ (in terms of applied electric field)

$v_d = \frac{I}{neA}$ (in terms of current through the conductor) From second relation

$I = neAv_d$ where A is the area of cross-section and "Av_d" represents the rate of flow.

The term $\frac{v_d}{E}$ is called mobility of charge carriers, represented by $\mu = \frac{v_d}{E} = \frac{e\tau}{m}$.

(here $\tau \rightarrow$ mean relaxation time depends on temperature $\tau \propto \frac{1}{\sqrt{T}}$, $T \rightarrow$ absolute temperature of the conductor)

OHM'S LAW

$I = \frac{V}{R}$ where $R = \frac{1}{\sigma}\frac{\ell}{A} = \frac{\rho\ell}{A}$ where ρ (resistivity) $= \frac{1}{\sigma}$

Hence according to Ohm's law when R is constant $I \propto V \Rightarrow I \sim V$ curve is a straight line (at constant temperature)

- Resistance of a conductor is given by

$$R = \frac{\rho\ell}{A} = \frac{m\ell}{ne^2\tau A}$$

where ρ is resistivity. Its units is Ω m.

- Resistivity of a conductor, $\rho = \frac{m}{ne^2\tau}$ (where m is mass of electron, n is number density of free electrons, τ is average relaxation time).

Variatio in resistance (R)

Variation with length: $R = \rho\frac{\ell}{A}$

(a) If a wire is cut to alter its length, then area remains same. $\therefore R \propto \ell$

(b) If a wire is stretched or drawn out or folded, area varies but volume remains constant. $\Rightarrow R \propto \ell^2$

For small percentage changes (< 5%) in length by stretching or folding, then, $\frac{\Delta R}{R} = \frac{2\Delta\ell}{\ell}$

Variation with area of cross-section or thickness

(a) If area is increased / decreased but length is kept same.

$\therefore R \propto \frac{1}{A}$ or $R \propto \frac{1}{r^2}$ (r = radius / thickness)

(b) If area is increased/decreased but volume remains same.

$$R \propto \frac{1}{A^2} \text{ or } R \propto \frac{1}{r^4}$$

For Conductors :

$\rho_t = \rho_0(1+\alpha t)$, where '$\alpha$' is temperature.

Coefficient of resistivity.

As $R \propto \rho \Rightarrow R = R_0(1 + \alpha t)$

(R_0 is the resistance at reference temperature)

At temperature t_1, $R_1 = R_0(1 + \alpha t_1)$

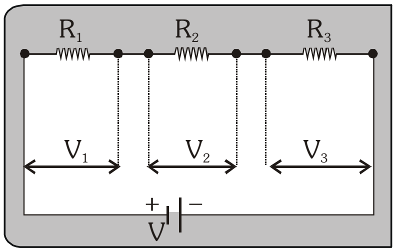
At temperature t_2, $R_2 = R_0(1 + \alpha t_2)$

$$\Rightarrow \alpha = \frac{R_2 - R_1}{R_0(t_2 - t_1)}, R_0 = \frac{R_2 - R_1}{\alpha(t_2 - t_1)}$$

COMBINATION OF RESISTANCES

Resistance In Series (Same Current)

$R = R_1 + R_2 + R_3 + \ldots\ldots\ldots\ldots\ldots + R_n$ and
$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots\ldots\ldots + V_n$.



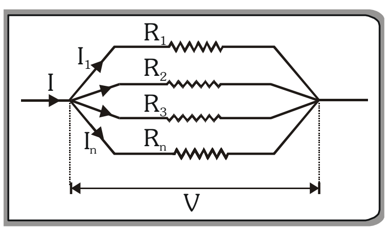
$\Rightarrow$ $R_{net} = R_1 + R_2 + R_3$

$\Rightarrow$ $V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3$

Resistance In Parallel (Same Potential difference)

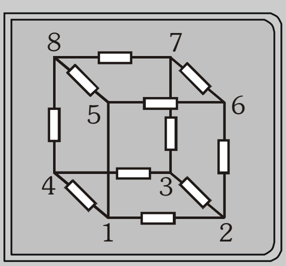
Effective resistance (R) then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots + \frac{1}{R_n}$$



For two resistance $R = \frac{R_1R_2}{R_1 + R_2}$

Equivalent Resistance In Cube (Symmatry)



(a) Resistance between two nearer corners

$R_{12} = \frac{7}{12}r$ $\quad C_{12} = \frac{12C}{7}$

(b) Resistance across face diagonal

$R_{13} = \frac{3}{4}r$ $\quad C_{13} = \frac{4C}{3}$

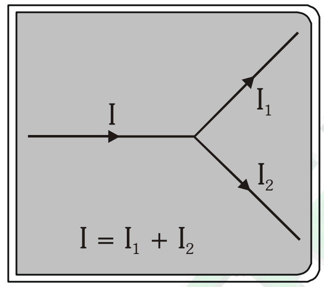
(c) Resistance across main diagonal

$R_{17} = \frac{5}{6}r$ $\quad C_{17} = \frac{6C}{5}$

KIRCHHOFF'S LAWS

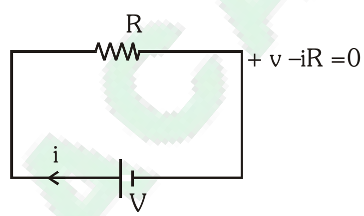
1. Junction Rule (K.C.L.)

It is based on conservation of charge.



2. Loop Rule (K.V.L.)

For any closed loop, total rise in potential + total fall in potential = 0.
It is based on conservation of energy.

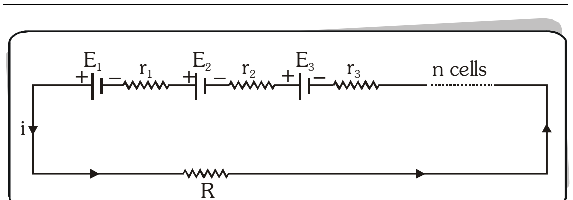


Terminal Voltage V = E – iR discharging,
E + iR charging

Cell

- **EMF (E)** : The potential difference across the terminals of a practical cell when no current is being drawn from it.
- **Internal Resistance (r)** : The opposition of flow of current inside the cell. It depends on
  (i) Distance between electrodes : ↑r↑
  (ii) Area of electrodes : ↑r↓
  (iii) Concentration of electrolyte : ↑r↑
  (iv) Temperature : ↑r↓

Series Compbination of Cells :



(a) $E_{equivalent} = E_1 + E_2 + E_3 + \ldots\ldots\ E_n$
(b) $r_{equivalent} = r_1 + r_2 + r_3 + \ldots\ldots\ r_n$

(c) Current $i = \frac{\sum E_i}{\sum r_i + R}$

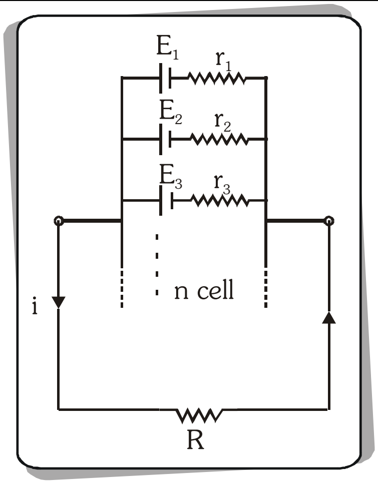
(d) If all cells have equal emf E and equal internal resistance r then $i = \frac{nE}{nr + R}$

Cases :

(i) If $nr >> R \Rightarrow i = \frac{E}{r}$

(ii) If $nr << R \Rightarrow i = \frac{nE}{R}$

Parallel Combination of cells :



(a) $E_{equivalent} = \dfrac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + ...}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ...}$

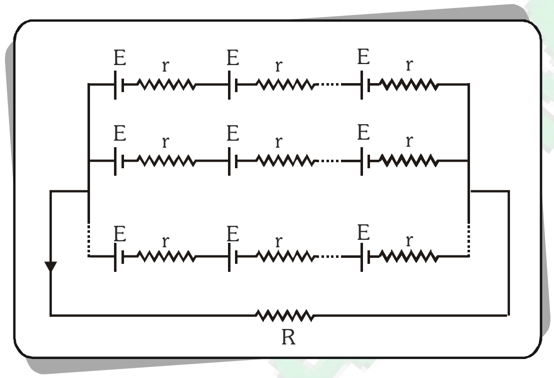
(b) $r_{equivalent} = \dfrac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ...}$

(c) If all cells have equal emf. E and internal resistance r then $E_{equivalent} = E$

$r_{equivalent} = \frac{r}{n} \Rightarrow$ current $i = \dfrac{E}{\frac{r}{n} + R}$

Mixed combination

Total number of identical cell in this circuit is nm. If n cells connected in a series and there are m such branches in the circuit than the internal resistance of the cells connected in a row = nr



Total internal resistance of the circuit $\frac{1}{r_{net}} = \frac{1}{nr} + \frac{1}{nr} +$ ....upto m turns

($\because$ There are such m rows) $r_{net} = \frac{nr}{m}$

Total e.m.f. of the circuit = total e.m.f. of the cells connected in a row $E_T = nE$

$$I = \frac{E_{net}}{R + r_{net}} = \frac{nE}{R + \frac{nr}{m}}$$

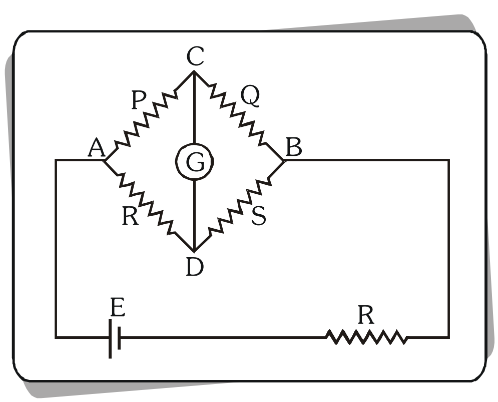
Current in the circuit is maximum when external resistance in the circuit is equal to the total internal resistance of the cells $R = \frac{nr}{m}$

WHEAT STONE BRIDGE

When current through the galvanometer is zero (null point or zero defluction) $\frac{P}{Q} = \frac{R}{S}$.

When $PS > QR, V_C < V_D$ & $PS < QR, V_C > V_D$ or $PS = QR \Rightarrow$ products of opposite arms are equal.

Potential difference between C & D at null point is zero . The null point is not affected by resistance of G & E. It is not affected even if the positions of G & E are interchanged.

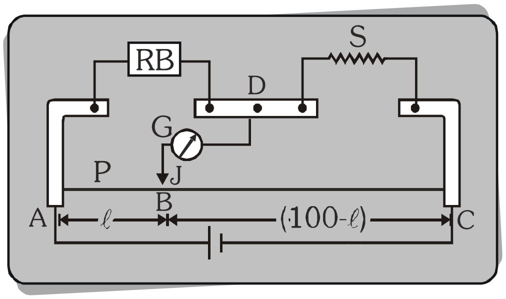


Electrical Instruments

Metre Bridge : Works on principle of wheet stone bridge

At balance condition :

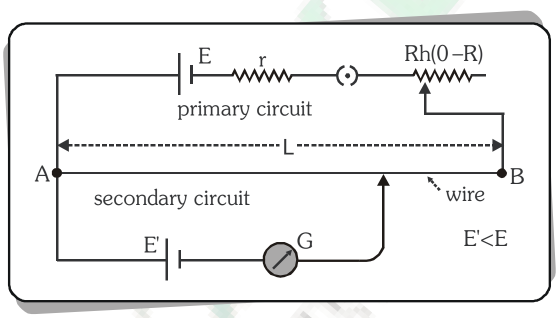
$$\frac{P}{Q}=\frac{R}{S}\Rightarrow\frac{\ell}{(100-\ell)}=\frac{R}{S}\Rightarrow S=\frac{(100-\ell)}{\ell}R$$



Potentiometer :

A potentiometer is a linear conductor of uniform cross-section with a steady current set up in it. This maintains a uniform potential gradient along the length of the wire . Any potential difference which is less than the potential difference maintained across the potentiometer wire can be measured using this .

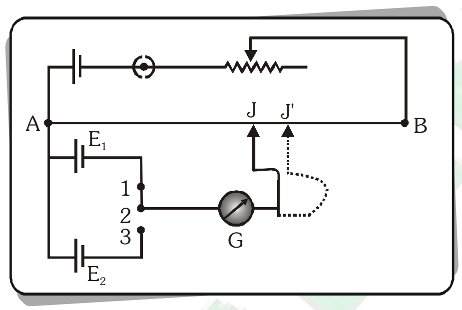
Circuits of potentiometer :



$$x=\frac{V}{L}=\frac{\text{current} \times \text{resistance of potentiometer wire}}{\text{length of potentiometer wire}}=I\left(\frac{R}{L}\right)$$

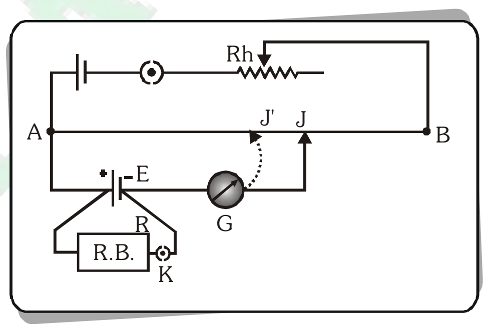
Applications of potentiometer :

(i) Comparision of emfs of two cells $\frac{E_1}{E_2}=\frac{\ell_1}{\ell_2}$



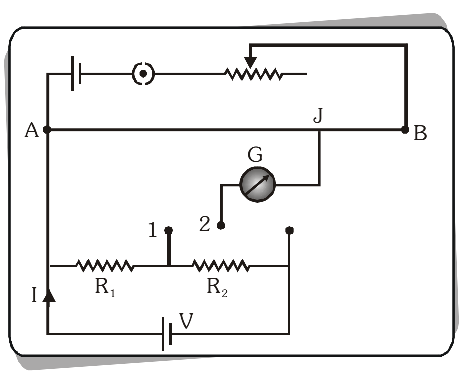
(ii) Internal Resistance of a given primary cell

$$r=\left(\frac{\ell_1-\ell_2}{\ell_2}\right)R$$



(iii) Comparision of two resistances

$$\frac{R_1}{R_1+R_2}=\frac{\ell_1}{\ell_2}$$



Galvanometer :

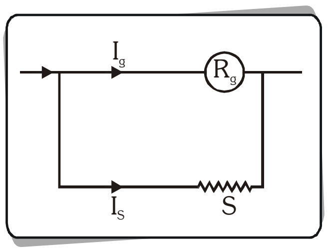
An instrument used to measure shtrength of current by measuring the diflection of me coil due to torque produced by a magnetic field.

$$T \propto i \propto \theta$$

A galvanometer can be converted into ammeter & voltmeter of varied scale as below.

Ammeter :

It is a modified form of suspended coil galvanometer, it is used to measure current. A shunt (small resistance) is connected in parallel with galvanometer to convert into ammeter .



$$S = \frac{I_g R_g}{I - I_g}$$

where

R_g = galvanometer resistance

I_g = Maximum current that can flow through the galvanometer .

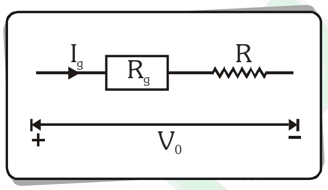
I = Maximum current that can be measured using the given ammeter .

An ideal ammeter has zero resistance.

Voltmeter :

A high resistance is put in series with galvanometer. It is used to measure potential difference.

$I_g = \frac{V_o}{R_g + R}$; $R \to \infty$, Ideal voltmeter



Electrical Power :

The energy liberated per second in a device is called its power. The electrical power P delivered by an electrical device is given by $P = VI$, where V = potential difference across device & I = current. If the current enters the higher potential point of the device then power is consumed by it (i.e. acts as load) . If the current enters the lower potential point then the device supplies power (i.e. acts as source). Power consumed by a resistor

$$P = I^2R = VI = \frac{V^2}{R}$$

Heating Effect Of Electric Current :

When a current is passed through a resistor energy is wasted in over coming the resistances of the wire. This energy is converted into heat

$$W = VIt = I^2Rt = \frac{V^2}{R}t$$

Joules Law Of Electrical Heating :

The heat generated (in joules) when a current of I ampere flows through a resistance of R ohm for T second is given by :

$$H = I^2RT \text{ joule} = \frac{I^2RT}{4.2} \text{ calories.}$$

If current is variable passing through the conductor then we use for heat produced in resistance in time 0 to T is:

$$H = \int_0^T I^2 R dt$$

Unit Of Electrical Energy Consumption :

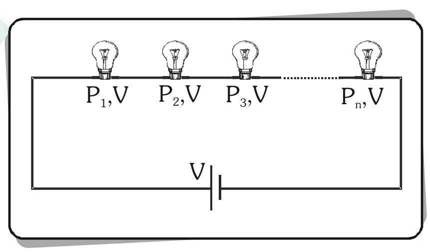
1 unit of electrical energy

= kilowatt hour

= 1 kWh = 3.6×10^6 joules.

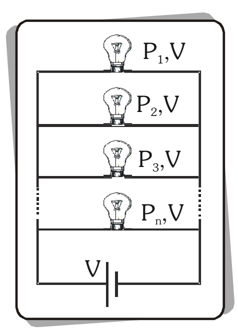
- **Series combination of Bulbs**

$$\frac{1}{P_{total}} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + ...$$



- **Parallel combination of Bulbs**

$P_{total} = P_1 + P_2 + P_3 + ...$



KEY POINTS

- A current flows through a conductor only when there is an electric field within the conductor because the drift velocity of electrons is directly proportional to the applied electric field.
- Electric field outside the conducting wire which carries a constant current is zero because net charge on a current carrying conductor is zero.
- A metal has a resistance and gets often heated by flow of current because when free electrons drift through a metal, they make occasional collisions with the lattice. These collisions are inelastic and transfer energy to the lattice as internal energy.
- Ohm's law holds only for small current in metallic wire, not for high currents because resistance increased with increase in temperature.
- Potentiometer is an ideal instrument to measure the potential difference because potential gradient along the potentiometer wire can be made very small.
- An ammeter is always connected in series whereas a voltmeter is connected in parallel because an ammeter is a low-resistance galvanometer while a voltmeter is a high-resistance galvanometer.
- Current is passed through a metallic wires, heating it red, when cold water is poured over half of the portion, rest of the portion becomes more hot because resistance decreases due to decrease in temperature so current through wire increases.