CAPACITANCE

CONCEPT OF CAPACITANCE

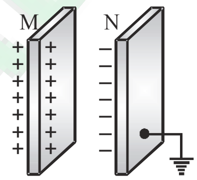
When a conductor is charged then its potential rises. The increase in potential is directly proportional to the charge given to the conductor. $Q \propto V \Rightarrow Q = CV$

The constant C is known as the capacity of the conductor.

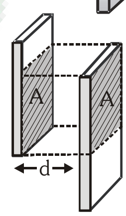
Capacitance is a scalar quantity with dimension $C = \frac{Q}{V} = \frac{Q^2}{W} = \frac{A^2T^2}{M^1L^2T^{-2}} = M^{-1}L^{-2}T^4A^2$

Unit :- farad, coulomb/volt

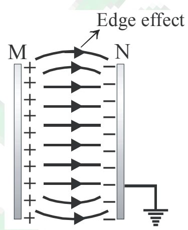
PARALLEL PLATE CAPACITOR



$$C = \frac{q}{V} = \frac{\varepsilon_r \varepsilon_0 A}{d}$$



- If one of the plates of parallel plate capacitor slides relatively than C decrease (As overlapping area decreases).
- If both the plates of parallel plate capacitor are touched each other resultant charge and potential became zero.
- Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.



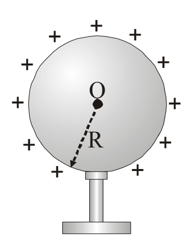
$\vec{E}$ = uniform in the centre

$\vec{E}$ = non-uniform at the edges

THE CAPACITANCE OF A SPHERICAL CONDUCTOR

When a charge Q is given to a isolated spherical conductor then its potential rises.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$

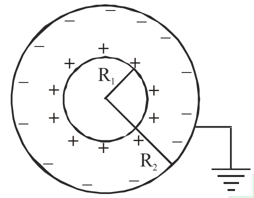


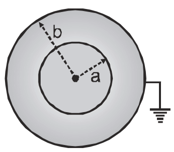
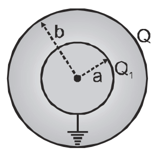
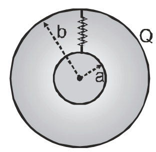
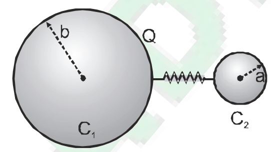
If conductor is placed in a medium then

$$C_{medium} = 4\pi\varepsilon R = 4\pi\varepsilon_0\varepsilon_r R$$

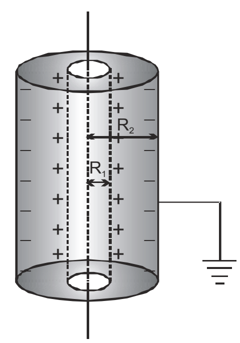
SPHERICAL CAPACITOR OUTER SPHERE IS EARTHED

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1} \text{ (in air or vacuum)}$$



Spherical capacitor outer is earthed	Inner is earthed and outer is given a charge	Connected and outer is given a charge	Connected spheres
			
$C = \frac{4\pi\varepsilon_0 ab}{b-a}$ (b > a)	$C = \frac{4\pi\varepsilon_0 b^2}{b-a}$ (b > a)	$C = 4\pi\varepsilon_0 b$	$C = C_1 + C_2$ $C = 4\pi\varepsilon_0(a+b)$

CYLINDRICAL CAPACITOR



Electrical field between cylinders $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{Q/\ell}{2\pi\varepsilon_0 r}$

Potential difference between plates $V = \int_{R_1}^{R_2} \frac{Q}{2\pi\varepsilon_0 r\ell} dr = \frac{Q}{2\pi\varepsilon_0 \ell} \ell n\left(\frac{R_2}{R_1}\right)$

Capacitance $C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 \ell}{\log_e(R_2/R_1)}$

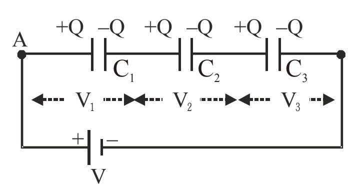
(ii) Force between the plates

$$F = -QE = -\frac{Q^2}{2\varepsilon_0 A}$$

Magnitude of force $F = \frac{Q^2}{2\varepsilon_0 A} = \frac{1}{2}\varepsilon_0 A E^2$

Force per unit area or energy density or electrostatic pressure $= \frac{F}{A} = u = p = \frac{1}{2}\epsilon_0 E^2$

COMBINATION OF CAPACITOR



- **Capacitor in series**:
  In this arrangement of capacitors the charge has no alternative path(s) to flow.

(i) The charges on each capacitor are equal
i.e. $Q = C_1V_1 = C_2V_2 = C_3V_3$

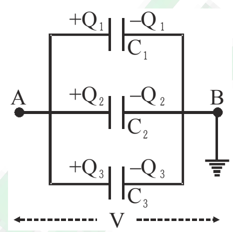
(ii) The total potential difference across AB is shared by the capacitors in the inverse ratio of the capacitances $V = V_1 + V_2 + V_3$

If C_S is the net capacitance of the series combination, then

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- **Capacitors in parallel**

In such arrangement of capacitors the charge has an alternative path(s) to flow.



(i) The potential difference across each capacitor is same and equal the total potential applied. i.e. $V = V_1 = V_2 = V_3 \Rightarrow V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$

(ii) The total charge Q is shared by each capacitor in the direct ratio of the capacitances. $Q = Q_1 + Q_2 + Q_3$

If C_P is the net capacitance for the parallel combination of capacitors :

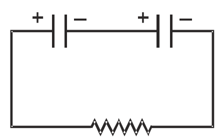
$$C_PV = C_1V + C_2V + C_3V \qquad \Rightarrow C_P = C_1 + C_2 + C_3$$

- For a given voltage to store maximum energy capacitors should be connected in parallel.
- If N identical capacitors each having breakdown voltage V are joined in

(i) series then the break down voltage of the combination is equal to NV

(ii) parallel then the breakdown voltage of the combination is equal to V.

- Two capacitors are connected in series with a battery. Now battery is removed and loose wires connected together then final charge on each capacitor is zero.



- If N identical capacitors are connected then $C_{series} = \frac{C}{N}$, $C_{parallel} = NC$

- In DC capacitor's offers infinite resistance in steady state, so there will be no current flows through capacitor branch.

ENERGY STORED IN A CHARGED CONDUCTOR/CAPACITOR

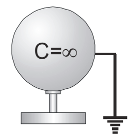
Let C is capacitance of a conductor. On being connected to a battery. It charges to a potential V from zero potential. If q is charge on the conductor at that time then q = CV. Let battery supplies small amount of charge dq to the conductor at constant potential V. Then small amount of work done by the battery against the force exerted by exsiting charge is

$$dW = Vdq = \frac{q}{C}dq \Rightarrow W = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q \Rightarrow W = \frac{Q^2}{2C}$$

where Q is the final charge acquired by the conductor. This work done is stored as potential energy, so

$$U = \frac{Q^2}{2C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV \quad \therefore U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

- As the potential of the Earth is assumed to be zero, capacity of earth or a conductor conncected to earth will be infinite $C = \frac{q}{V} = \frac{q}{0} = \infty$



- Actual capacity of the Earth $C = 4\pi\varepsilon_0 R = \frac{1}{9\times10^9}\times 64\times10^5 = 711\ \mu F$
- Work done by battery W_b = (charge given by battery) × (emf) = QV but

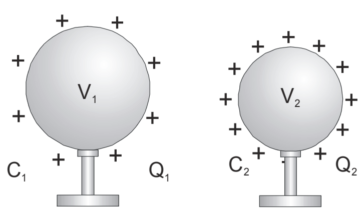
  Energy stored in conductor $= \frac{1}{2}QV$

  so 50% energy supplied by the battery is lost in form of heat.

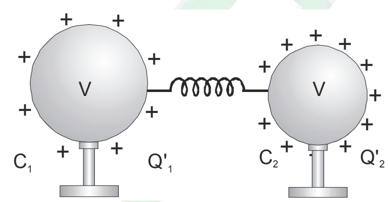
REDISTRIBUTION OF CHARGES AND LOSS OF ENERGY

When two charged conductors are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential. This flow of charge stops when the potential of two conductors became equal.

Let the amounts of charges after the conductors are connected are Q_1' and Q_2' respectively and potential is V then



(Before connection)



(After connection)

- **Common potential**

  According to law of Conservation of charge $\quad Q_{\text{before connection}} = Q_{\text{after connection}}$

  $\Rightarrow C_1V_1 + C_2V_2 = C_1V + C_2V$

  Common potential after connection $\boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$

- **Charges after connection**

  $$Q_1' = C_1V = C_1\left(\frac{Q_1+Q_2}{C_1+C_2}\right) = \left(\frac{C_1}{C_1+C_2}\right)Q$$ (Q : Total charge on system)

  $$Q_2' = C_2V = C_2\left(\frac{Q_1+Q_2}{C_1+C_2}\right) = \left(\frac{C_2}{C_1+C_2}\right)Q$$

  Ratio of the charges after redistribution $\boxed{\frac{Q_1'}{Q_2'} = \frac{C_1V}{C_2V} = \frac{R_1}{R_2} \text{ (in case of spherical conductors)}}$

- **Loss of energy in redistribution**

  When charge flows through the conducting wire then **energy is lost mainly on account of Joule effect**, electrical energy is converted into heat energy, so change in energy of this system,

  $$\Delta U = U_f - U_i \Rightarrow \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2\right) - \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) \Rightarrow \Delta U = -\frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1 - V_2)^2$$

  Here negative sign indicates that energy of the system decreases in the process.

EFFECT OF DIELECTRIC

- The insulators in which microscopic local displacement of charges takes place in presence of electric field are known as **dielectrics**.
- Dielectrics are non conductors upto certain value of field depending on its nature. If the field exceeds this limiting value called **dielectric strength** they lose their insulating property and begin to conduct.
- **Dielectric strength** is defined as the maximum value of electric field that a dielectric can tolerate without breakdown. Unit is volt/metre. Dimensions $M^1L^1T^{-3}A^{-1}$

Polar dielectrics

- In absence of external field the centres of positive and negative charge do not coincide-due to asymmetric shape of molecules.
- Each molecule has permanent dipole moment.
- The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.
- In presence of external field dipoles tends to align in direction of field.

  Ex. Water, Alcohol, CO_2, $HC\ell$, NH_3

Non polar dielectrics

- In absence of external field the centre of positive and negative charge coincides in these atoms or molecules because they are symmetric.
- The dipole moment is zero in normal state.
- In presence of external field they acquire induced dipole moment.

  Ex. Nitrogen, Oxygen, Benzene, Methane

Polarisation :

The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarisation.

- **Polarisation vector** $\vec{P}$

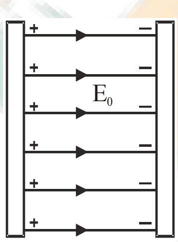
  This is a vector quantity which describes the extent to which molecules of dielectric become polarized by an electric field or oriented in direction of field.

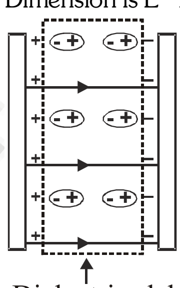
  $\vec{P}$ = the dipole moment per unit volume of dielectric = $n\vec{p}$

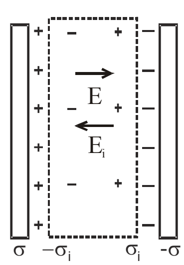
  where n is number of atoms per unit volume of dielectric and $\vec{p}$ is dipole moment of an atom or molecule.

  $$\vec{P} = n\vec{p} = \frac{q_i d}{Ad} = \left(\frac{q_i}{A}\right) = \sigma_i = \text{induced surface charge density.}$$

  Unit of $\vec{P}$ is C/m^2 Dimension is $L^{-2}T^1A^1$







Dielectric slab

Let E_0, V_0, C_0 be electric field, potential difference and capacitance in absence of dielectric. Let E, V, C are electric field, potential difference and capacitance in presence of dielectric respectively.

Electric field in absence of dielectric $E_0 = \dfrac{V_0}{d} = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0 A}$

Electric field in presence of dielectric $E = E_0 - E_i = \dfrac{\sigma - \sigma_i}{\varepsilon_0} = \dfrac{Q - Q_i}{\varepsilon_0} = \dfrac{V}{d}$

Capacitance in absence of dielectric $C_0 = \dfrac{Q}{V_0}$

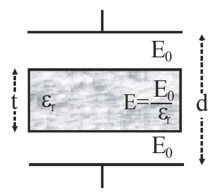
Capacitance in presence of dielectric $C = \dfrac{Q - Q_i}{V}$

The dielectric constant or relative permittivity K

or $\varepsilon_r = \dfrac{E_0}{E} = \dfrac{V_0}{V} = \dfrac{C}{C_0} = \dfrac{Q}{Q - Q_i} = \dfrac{\sigma}{\sigma - \sigma_i} = \dfrac{\varepsilon}{\varepsilon_0}$

From $K = \dfrac{Q}{Q - Q_i} \Rightarrow Q_i = Q\left(1 - \dfrac{1}{K}\right)$ and $K = \dfrac{\sigma}{\sigma - \sigma_i} \Rightarrow \sigma_i = \sigma\left(1 - \dfrac{1}{K}\right)$

If capacitor is partially filled with dielectric

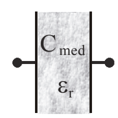


$V = E_0(d - t) + E t$

$\Rightarrow \quad V = E_0\left[d - t + \left(\dfrac{E}{E_0}\right)t\right] \quad \because \dfrac{E_0}{E} = \varepsilon_r = \text{Dielectric constant}$

$\Rightarrow V = \dfrac{\sigma}{\varepsilon_0}\left[d - t + \dfrac{t}{\varepsilon_r}\right] = \dfrac{q}{A\varepsilon_0}\left[d - t + \dfrac{t}{\varepsilon_r}\right] \Rightarrow C = \dfrac{q}{V} = \dfrac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{\varepsilon_r}\right)} = \dfrac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{\varepsilon_r}\right)}$ ...(i)

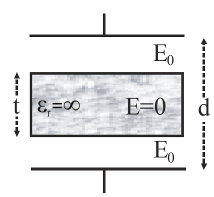
If medium is fully present between the space.



$\because \quad t = d$

Now from equation (i) $\quad C_{medium} = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$

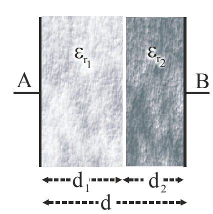
If capacitor is partialy filled by a conducting slab of thickness (t< d).



$\because \quad \varepsilon_r = \infty$ for conductor $C = \dfrac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{\infty}\right)} = \dfrac{\varepsilon_0 A}{(d - t)}$

DISTANCE AND AREA DIVISION BY DIELECTRIC

- **Distance Division**



(i) Distance is divided and area remains same.

(ii) Capacitors are in series.

(iii) Individual capacitances are $C_1 = \dfrac{\varepsilon_0 \varepsilon_{r_1} A}{d_1}$, $C_2 = \dfrac{\varepsilon_0 \varepsilon_{r_2} A}{d_2}$

These two are in series $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} \Rightarrow \dfrac{1}{C} = \dfrac{d_1}{\varepsilon_0 \varepsilon_{r_1} A} + \dfrac{d_2}{\varepsilon_0 \varepsilon_{r_2} A}$

$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 A}\left[\frac{d_1\varepsilon_{r_2} + d_2\varepsilon_{r_1}}{\varepsilon_{r_1}\varepsilon_{r_2}}\right] \Rightarrow C = \varepsilon_0 A\left[\frac{\varepsilon_{r_1}\varepsilon_{r_2}}{d_1\varepsilon_{r_2} + d_2\varepsilon_{r_1}}\right]$$

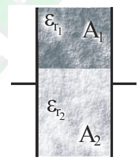
Special case : If $d_1 = d_2 = \frac{d}{2}$ $\Rightarrow C = \frac{\varepsilon_0 A}{d}\left[\frac{2\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2}}\right]$

- **Area Division**

(i) Area is divided and distance remains same.

(ii) Capacitors are in parallel.

(iii) Individual capacitances are $C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} A_1}{d}$ $C_2 = \frac{\varepsilon_0 \varepsilon_{r_2} A_2}{d}$



These two are in parallel so $C = C_1 + C_2 = \frac{\varepsilon_0 \varepsilon_{r_1} A_1}{d} + \frac{\varepsilon_0 \varepsilon_{r_2} A_2}{d} = \frac{\varepsilon_0}{d}(\varepsilon_{r_1} A_1 + \varepsilon_{r_2} A_2)$

Special case : If $A_1 = A_2 = \frac{A}{2}$ Then $C = \frac{\varepsilon_0 A}{d}\left(\frac{\varepsilon_{r_1} + \varepsilon_{r2}}{2}\right)$

- **Variable Dielectric Constant :**

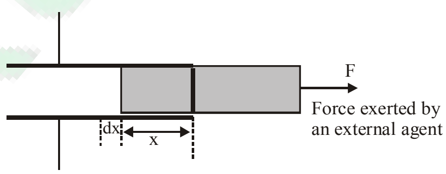
If the dielectric constant is variable, then equivalent capacitance can be obtained by selecting an element as per the given condition and then integrating.

(i) If different elements are in parallel, then $C = \int dC$, where dC = capacitance of selected differential element.

(ii) If different element are in series, then $\frac{1}{C} = \int d\left(\frac{1}{C}\right)$ is solved to get equivalent capacitance C.

FORCE ON A DIELECTRIC IN A CAPACITOR

Consider a differential displacement dx of the dielectric as shown in figure always keeping the net force on it zero so that the dielectric moves slowly without acceleration. Then, $W_{Electrostatic} + W_F = 0$, where W_F denotes the work done by external agent in displacement dx



$W_F = -W_{Electrostatic}$ $W_F = \Delta U$

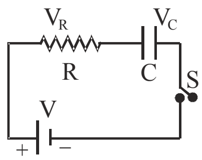
$$\Rightarrow -F.dx = \frac{Q^2}{2} d\left[\frac{1}{C}\right] \left[U = \frac{Q^2}{2C}\right] \Rightarrow -F.dx = \frac{-Q^2}{2C^2} dC \Rightarrow F = \frac{Q^2}{2C^2}\left(\frac{dC}{dx}\right)$$

This is also true for the force between the plates of the capacitor. If the capacitor has battery connected to it, then as the p.d. across the plates is maintained constant. $V = \frac{Q}{C} \Rightarrow F = \frac{1}{2}V^2 \frac{dC}{dx}$.

CHARGING & DISCHARGING OF A CAPACITOR

Charging

- When a capacitor, resistance, battery, and key is conected in series and key is closed, then



- **Charge at any instant**

$$V = V_C + V_R = \frac{Q}{C} + IR = \frac{Q}{C} + \frac{dQ}{dt}R$$

$$Q = CV\left[1 - e^{-t/RC}\right] = Q_0\left[1 - e^{-t/RC}\right]$$

At $t = \tau = RC$ = time constant

$Q = Q_0[1 - e^{-1}] = 0.632\,Q_0$

So, in charging, charge increases to 63.2% of charge in the time equal to τ.

- **Current at any instant**

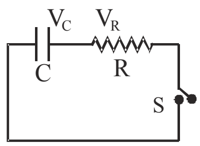
$i = dQ/dt = i_0 e^{-t/RC}$ $\{i_0 = Q_0/RC\}$

- **Potential at any instant**

$V = V_0(1 - e^{-t/RC})$

Discharging

- When a charged capacitor, resistance and keys is conected in series and key is closed. Then energy stored in capacitor is used to circulate current in the circuit.



- **Charge at any instant**

$V_C + V_R = 0$

$Q = Q_0 e^{-t/RC}$

At $t = \tau = RC$ = time constant

$Q = Q_0 e^{-1} = 0.368\,Q_0$

So, in discharging, charge decreases to 36.8% of the initial charge in the time equal to τ.

- **Current at any instant**

$i = dQ/dt = -i_0 e^{-t/RC}$ $\{i_0 = Q_0/RC\}$

- **Potential at any instant**

$V = V_0\, e^{-t/RC}$