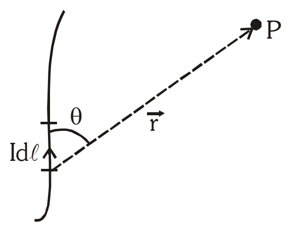
Magnetic Effect of Current & Magnetism

Magnetic effect of current discovered by : Orested

BIOT-SAVART'S LAW :-

→ The magnetic field dB at a point due to current element $I\vec{d\ell}$ is given by ,

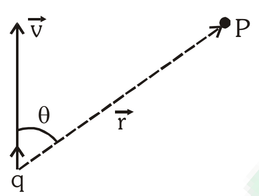


$I\vec{d\ell}$ is given by

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{(Id\vec{\ell} \times \vec{r})}{r^3}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{Id\ell \sin\theta}{r^2}$$

→ Magnetic field at P due to moving charge is given by,

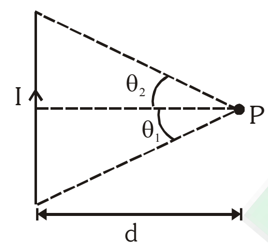


$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v} \times \vec{r})}{r^3}$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$$

1. Magnetic field due to finite current carrying wire at point P,

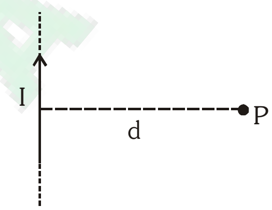
$$B_P = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$



(a) For infinite wire,

$\theta_1 = \theta_2 = 90°$

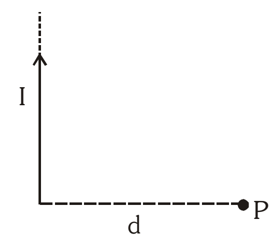
$$B_P = \frac{\mu_0 I}{2\pi d}$$



(b) For semi-infinite wire,

$\theta_1 = 0°,\ \theta_2 = 90°$

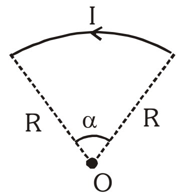
$$B_P = \frac{\mu_0 I}{4\pi d}$$



Note : For points along the length of the wire (but not on it), the field is always zero.

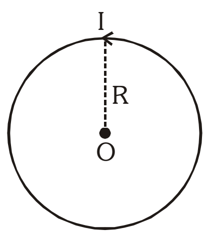
2. Magnetic field at the centre of current carrying circular arc.

$$B_0 = \frac{\mu_0 I}{4\pi R}(\alpha)$$



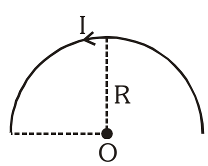
(a) At the centre of current carrying circular loop,

$$B_0 = \frac{\mu_0 I}{2R}$$



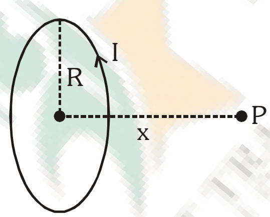
(b) At the centre of semi-circular arc

$$B_0 = \frac{\mu_0 I}{4R}$$



3. Magnetic-field at an axial point of current carrying circular loop,

$$B_P = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}$$



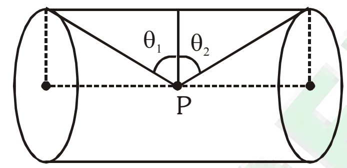
4. Magnetic field at the axis of solenoid :

(a) Finite length :

$$B_P = \frac{\mu_0 n I}{2}\left[\sin\theta_1 + \sin\theta_2\right]$$

(b) Infinite length :

$$B_P = \mu_0 n I$$



$n \rightarrow$ number of turns per unit length.

Note : Magnetic field outside solenoid is zero.

AMPERE'S LAW :

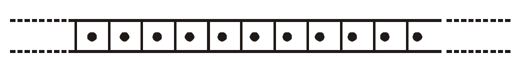
The line integral of magnetic field over the closed path $\left(\oint \vec{B}\cdot\vec{d\ell}\right)$ is equal to μ_0 times the net current crossing the area inclosed by the path.

$$\oint \vec{B}\cdot\vec{d\ell} = \mu_0 I_{enclosed}$$

Here $\vec{B}$ is net magnetic field.

(i) Magnetic field due to infinite current sheet.

$$B = \frac{\mu_0 k}{2}$$



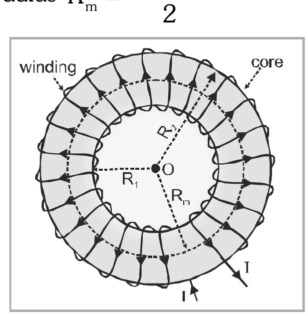
Here k is linear current density

(ii) Magnetic field inside toroid :

Field inside toroid :-

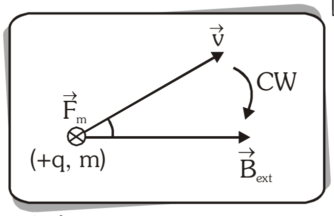
$B = \mu_0 n I$, where $n = N/2\pi R_m$, turn density

mean radius $R_m = \dfrac{R_1 + R_2}{2}$



Magnetic force on moving charge in magnetic field

Vector from $\vec{F}_m = q(\vec{v} \times \vec{B}_{ext})$ Always $\begin{bmatrix} \vec{F}_m \perp \vec{v} \\ \vec{F}_m \perp \vec{B}_{ext} \end{bmatrix}$

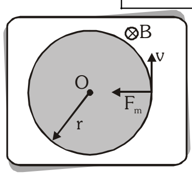


Magnitude form :

$F_m = qvB\sin\theta$ — $\theta = 90°$ ($\vec{v} \perp \vec{B}$) $\Rightarrow$ $\boxed{F_m = qvB}_{(max)}$; $\theta = 0°$ or $180° \Rightarrow \boxed{F_m = 0}_{(min)}$

Motion of charge in uniform field

$(\vec{v} \perp \vec{B},\ \theta = 90°)$ $\boxed{qvB = \frac{mv^2}{r}}$



(a) Radius of circular path :

$r = \frac{mv}{qB}$, where $P = mv = \sqrt{2mE_K} = \sqrt{2mqV_{acc}}$

(b) Time period : $T = \frac{2\pi m}{qB}$

(c) Kinetic energy of charge : $E_K = \frac{(qBr)^2}{2m}$

Motion of charge in uniform field at any angle except 0° or 180° or 90°

(a) Radius of helical path : $r = \frac{mv\sin\theta}{qB}$

(b) Time period : $T = \frac{2\pi m}{qB}$

(c) Pitch of helix : $P = (v\cos\theta)\,T$, where $T = \frac{2\pi m}{qB}$

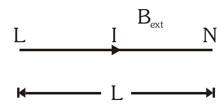
Combined effect of $\vec{E}$ & $\vec{B}$ on moving charge

Electromegnetic or Lorentz force

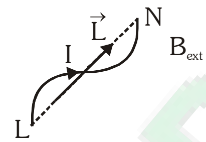
$\vec{F}_L = \vec{F}_e + \vec{F}_m$ $\boxed{\vec{F}_L = q\vec{E} + q(\vec{v} \times \vec{B})}$

Magnetic force on current carrying wire (or conductor)

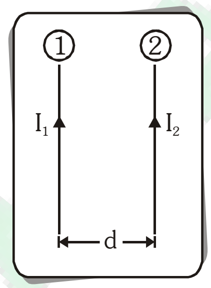
(a) Straight wire :- $\vec{F}_m = I(\vec{L} \times \vec{B}_{ext/uniform})$



(b) Arbitrary wire :- $\vec{F}_m = I(\vec{L} \times \vec{B}_{ext/uniform})$



Magnetic force b/w two long parallel wires



$f = \frac{\mu_0 I_1 I_2}{2\pi d}$ N/m

$\begin{bmatrix} \text{parallel currents} \Rightarrow \text{Attraction} \\ \text{antiparallel currents} \Rightarrow \text{Repulsion} \end{bmatrix}$

MAGNETIC TORQUE ON A CLOSED CURRENT CIRCUIT

When a plane closed current circuit is placed in uniform magnetic field , it experience a zero net force, but experience a torque given by $\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{M} \times \vec{B} = BINA \sin\theta$ where $\vec{A}$ = area vector outward from the face of the circuit where the current is anticlockwise, $\vec{B}$ = magnetic induction of the uniform magnetic field. $\vec{M}$ = magnetic moment of the current circuit = $IN\vec{A}$

Note : This expression can be used only if $\vec{B}$ is uniform.

Moving Coil Galvanometer

It consists of a plane coil of many turns suspended in a radial magnetic field. When a current is passed in the coil it experiences a torque which produces a twist in the suspension.

This deflection is directly proportional to the torque

$\therefore NIAB = K\theta$;

$$I = \left(\frac{K}{NAB}\right)\theta;$$

K=elastic torsional constant of the suspension

$$I = C\theta; \quad C = \frac{K}{NAB}$$

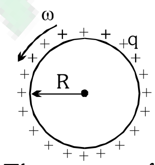
= Galvanometer constant

Magnetic dipole

- Magnetic moment $M = m \times 2\ell$, where m is pole strength of the magnet
- Magnetic field at axial point (or End-on position) of dipole $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{r^3}$
- Magnetic field at equitorial position (Broad-side on position) of dipole $$\vec{B} = \frac{\mu_0}{4\pi}\frac{(-\vec{M})}{r^3}$$
- Magnetic field at a point which is at a distance r from dipole midpoint and making angle θ with dipole axis. $$B = \frac{\mu_0}{4\pi}\frac{M\sqrt{1+3\cos^2\theta}}{r^3}$$
- Torque on dipole placed in uniform magnetic field $\vec{\tau} = \vec{M} \times \vec{B}$
- Potential energy of dipole placed in an uniform field $U = -\vec{M}\cdot\vec{B}$

Magnetic moment of a rotating charge

If a charge q is rotating at an angular velocity ω, its equivalent current is given as $I = \frac{q\omega}{2\pi}$ & its magnetic moment is $M = I\pi R^2 = \frac{1}{2}q\omega R^2$.



NOTE: The ratio of magnetic moment to angular momentum called gyromagnetic ratio of a uniform rotating object which is charged uniformly is always a constant and equal to half of specific charge. Irrespective of the shape of conductor $\boxed{M/L = q/2m}$

GILBERT'S MAGNETISM (EARTH'S MAGNETIC FIELD)

(a) Imaginary vertical plane passing through the magnetic North - South poles at that place. This plane is called the **MAGNETIC MERIDIAN**. The Earth's Magnetic poles are opposite to the geometric poles i.e. at earth's north pole, its geomagnetic south pole is situated and vice versa.

(b) On the magnetic meridian, the magnetic induction vector of the earth at any point, generally inclined to the horizontal at an angle called the **MAGNETIC DIP** at that place , such that $\vec{B}$ = total magnetic induction of the earth at that point.

$\vec{B}_v$ = the vertical component of $\vec{B}$ in the magnetic meridian plane = B sin θ

$\vec{B}_H$ = the horizontal component of $\vec{B}$ in the magnetic meridian plane=$B\cos\theta$. $\frac{B_v}{B_H} = \tan\theta$

(c) At a given place on the surface of the earth , the magnetic meridian and the geographic meridian may not coincide . The angle between them is called "**DECLINATION AT THAT PLACE**"

- **Intensity of magnetisation** $I = M/V$
- **Magnetic induction** $B = \mu H = \mu_0(H + I)$
- **Magnetic permeability** $\mu = \frac{B}{H}$
- **Magnetic susceptibility** $\chi_m = \frac{I}{H} = \mu_r - 1$
- **Curie law**
  - For paramagnetic materials $\chi_m \propto \frac{1}{T}$
- **Curie Wiess law**
  - For Ferromagnetic materials $\chi_m \propto \frac{1}{T - T_C}$

    Where T_C = curie temperature

KEY POINTS

- A charged particle moves perpendicular to magnetic field. Its kinetic energy will remain constant but momentum changes because magnetic force acts perpendicular to velocity of particle.
- If a unit north pole rotates around a current carrying wire then work has to be done because magnetic field produced by current is always non-conservative in nature.
- In a conductor, free electrons keep on moving but no magnetic force acts on a conductor in a magnetic field because in a conductor, the average thermal velocity of electrons is zero.
- Magnetic force between two charges is generally much smaller than the electric force between them because speeds of charges are much smaller than the free space speed of light.

Note : $\frac{F_{magnetic}}{F_{electric}} = \frac{v^2}{c^2}$