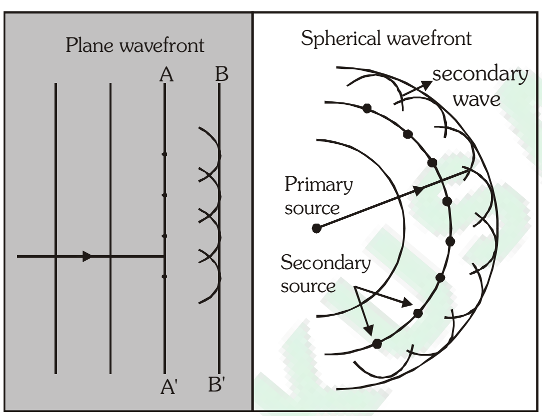
Wave Nature of Light & Wave Optics

HUYGEN'S WAVE THEORY

Huygen's in 1678 assumed that a body emits light in the form of waves.

- Each point source of light is a centre of disturbance from which waves spread in all directions. The locus of all the particles of the medium vibrating in the same phase at a given instant is called a wavefront.
- Each point on a wave front is a source of new disturbance, called secondary wavelets. These wavelets are spherical and travel with speed of light in that medium.
- The forward envelope of the secondary wavelets at any instant gives the new wavefront.
- In homogeneous medium, the wave front is always perpendicular to the direction of wave propagation.



COHERENT SOURCES :

Two sources will be coherent if and only if they produce waves of same frequency (and hence wavelength) and have a constant initial phase difference.

INCOHERENT SOURCES :

Two sources are said to be incoherent if they have different frequency and initial phase difference is not constant w.r.t. time.

INTERFERENCE : YDSE

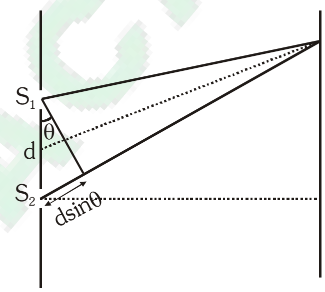
- Resultant intensity for coherent sources

  $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi_0$

- Resultant intensity for incoherent sources $I = I_1 + I_2$
- Intensity $\propto$ width of slit $\propto$ (amplitude)^2

  $\Rightarrow \frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{a_1^2}{a_2^2} \Rightarrow \frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2$

- Distance of n^{th} bright fringe $x_n = \frac{n\lambda D}{d}$



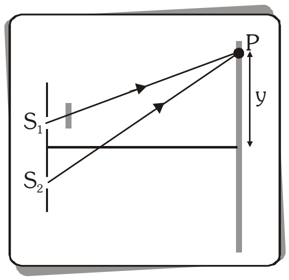
Path difference = $n\lambda$ where n =0, 1, 2, 3, ...

- Distance of m^{th} dark fringe $x_m = \frac{(2m+1)\lambda D}{2d}$

  Path difference=$(2m+1)\frac{\lambda}{2}$ where m= 0,1,2, 3,.....

- Fringe width $\beta = \frac{\lambda D}{d}$
- Angular fringe width $= \frac{\beta}{D} = \frac{\lambda}{d}$
- Fringe visibility $= \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100\,\%$
- If a transparent sheets of referactive index μ and thickness t is introduced in one of the paths of interfering waves, optial path will becomes 'μt' instead of 't'. Entire fringe pattern is displaced by $\frac{D\left[(\mu-1)t\right]}{d} = \frac{\beta}{\lambda}(\mu-1)t$ towards the side in which the thin sheet is introduced without any change in fringe width.

SHIFTING OF FRINGES



- Path difference produced by a slab $\Delta x = (\mu - 1)t$
- Fringe shift, $\Delta x = \frac{\beta}{\lambda}(\mu - 1)t = \frac{D}{d}(\mu - 1)t$
- Number of fringes shift

$$= \frac{\text{shift}}{\text{fringe width}} = \frac{t(\mu-1)D/d}{\lambda D/d} = \frac{(\mu-1)t}{\lambda}$$

INTEREFERENCE IN THE FILM

- For reflected Light :

  Maxima $\rightarrow 2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

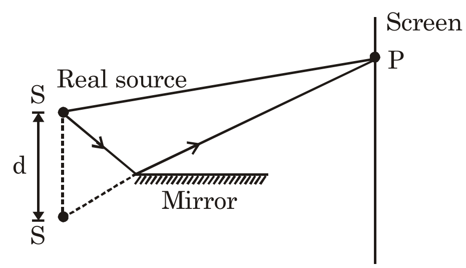
  Minima $\rightarrow 2\mu t \cos r = n\lambda$
- For transmitted light :

  Maxima $\rightarrow 2\mu t \cos r = n\lambda$

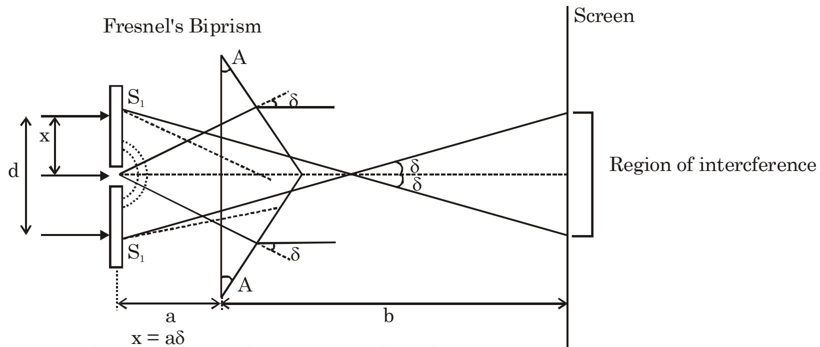
  Minima $\rightarrow 2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

  (t =thickness of film, μ=R.I. of the film)

- **Newton's Ring** : When a lens of large Radius of curvature is placed on a plane glass plate, an air film is formed between lower surface of the lens and the upper surface of the plate. It this film is illuminated by sodium light, due to interference concentric bright and dark rings called Newton's Rings are seen.
- If the film is wedge shaped. The fringes will be straight lines parallel to the edge of apex with minimum at the apex and fringe width $\beta = \frac{\lambda}{2\mu\theta}$ where $\theta = \frac{t}{x}$.
- If case of Newton's rings the centre is dark spacing between rings goes on decreasing as we move away from centre and radius of dark rings is proportional to the square root of all natural number while bright rings 1.0 the square root of odd numbers.
- **Llayd's Mirror** :



The position of dark and bright fringes are reversed relative to the pattern of two real sources because there is a 180^0 phase change produced by reflection.

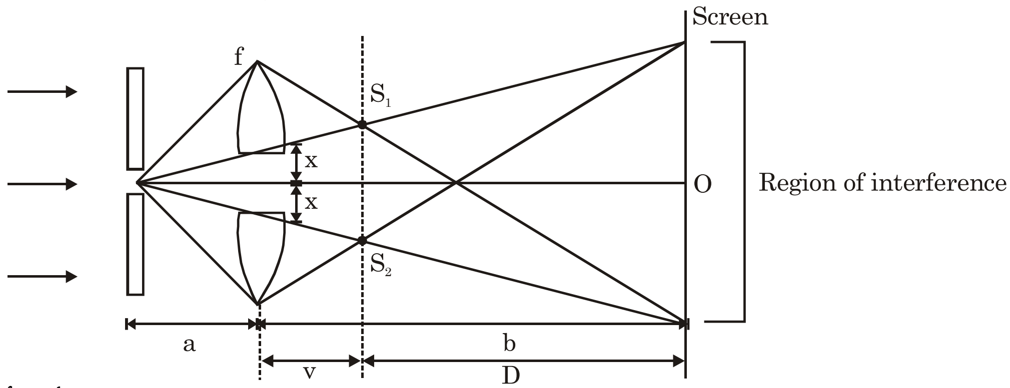


Seperation between coherent sources $d = 2a\delta = 2aA(\mu - 1)$

Seperation between slit plane and screen $\Delta = a + b$

Fringe width on screen $\beta = \dfrac{AD}{d} = \dfrac{\lambda(a+b)}{2aA(\mu-1)}$

Frillet split lens as a limiting case of YDSE



from lens

formula $v = \dfrac{af}{a-f}$

$D = a + b - |V|$

$d = 2x + 2\left|\dfrac{v}{u}\right|x$

$d = 2x\left(1 + \left|\dfrac{v}{u}\right|\right)$

Fringe width $\beta = \dfrac{\lambda D}{d}$

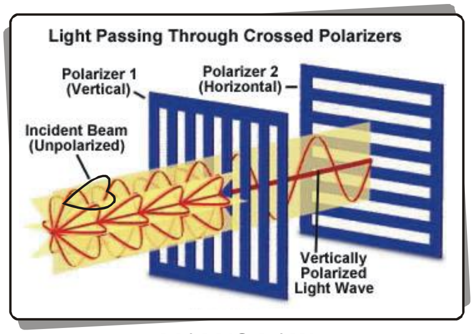
DIFFRACTION

- **Fresnel's diffraction** : In Fresnel's diffraction, the source and screen are placed close to the aperture or the obstacle and light after diffraction appears converging towards the screen and hence no lens is required to observed it. The incident wave fronts are either spherical or cylindrical.
- **Fraunhofer's diffraction :** The source and screen are placed at large distances from the aperture or the obstacle and converging lens is used to observed the diffraction pattern. The incident wavefront is planar one.
  - ❒ For minima : $a \sin\theta_n = n\lambda$
  - ❒ For maxima : $a \sin\theta_n = (2n+1)\dfrac{\lambda}{2}$
  - ❒ Linear width of central maxima : $W_x = \dfrac{2\lambda D}{a}$
  - ❒ Angular width of central maxima $W_\theta = \dfrac{2\lambda}{a}$
  - ❒ Intensity of maxima
    where I_0 = Intensity of central maxima

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 \text{ and } \beta = \frac{2\pi}{\lambda}a\sin\theta$$

POLARISATION OF LIGHT

If the vibrations of a wave are present in just one direction in a plane perpendicular to the direction of propagation, the wave is said to be **polarised or plane polarised**. The phenomenon of restricting the oscillations of a wave to just one direction in the transverse plane is called polarisation of waves.



MALUS LAW

The intensity of transmitted light passed through an analyser is $I=I_0\cos^2\theta$

(θ=angle between transmission directions of polariser and analyser)

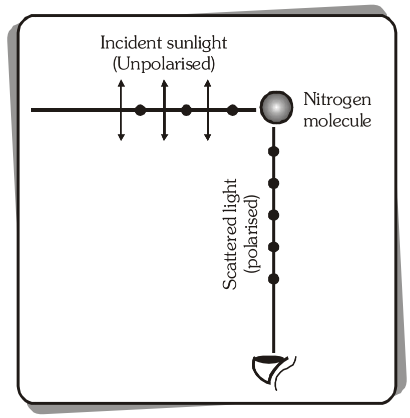
POLARISATION BY REFLECTION

Brewster's Law : The tangent of polarising angle of incidence at which reflected light becomes completely plane polarised is numerically equal to refractive index of the medium. $\mu=\tan i_p$;

i_p =Brewster's angle and $i_p+r_p=90°$

POLARISATION BY SCATTERING

If we look at the blue portion of the sky through a polaroid and rotate the polaroid, the transmitted light shows rise and fall of intensity.



The scattered light screen in a direction perpendicular to the direction of incidence is found to be plane polarised.

KEY POINTS

- The law of conservation of energy holds good in the phenomenon of interference.
- Fringes are neither image nor shadow of slit but locus of a point which moves such a way that its path difference from the two sources remains constant.
- In YDSE the interference fringes for two coherent point sources are hyperboloids with axis S_1S_2.
- If the interference experiment is repeated with bichromatic light, the fringes of two wavelengths will be coincident for the first time when

$$n(\beta)_{longer}=(n+1)(\beta)_{shorter}$$

- No interference pattern is detected when two coherent sources are infinitely close to each other, because $\beta \propto \frac{1}{d}$
- If maximum number of maximas or minimas are asked in the question, use the fact that value of $\sin\theta$ or $\cos\theta$ can't be greater than 1. $n_{max}=\frac{d}{\lambda}$ Total maxima $=2n_{max}+1$
- **Limit of resolution for microscope** $=\frac{1.22\lambda}{2a\sin\theta}=\frac{1}{\text{resolving power}}$
- **Limit of resolution for telescope** $=\frac{1.22\lambda}{a}=\frac{1}{\text{resolving power}}$